

$$y = \frac{x^2 + x + y}{x - 1} \rightarrow yx - y = x^2 + x + y \rightarrow x^2 + x(1 - y) + y + y = 0 \rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\rightarrow x = \frac{-(1 - y) \pm \sqrt{(1 - y)^2 - (4)(1)(y + y)}}{2} \rightarrow \Delta \geq 0 \rightarrow (1 - y)^2 - (4)(1)(y + y) \geq 0$$

$$\rightarrow 1 + y^2 - 4y - 4 - 4y \geq 0 \rightarrow y^2 - 8y - 3 \geq 0 \rightarrow (y - 4)(y + 1) \geq 0 \rightarrow \begin{matrix} - & + & - & + \\ + & - & + & - \end{matrix}$$

$$\rightarrow R_f = (-\infty, -1] \cup [3, +\infty)$$

الف) $y = 2x^2 - 5x + 1 \rightarrow \text{ext} \left| \begin{matrix} -\frac{b}{2a} = \frac{5}{4} \\ -\frac{1}{4} \end{matrix} \right| \rightarrow R_f = \left[-\frac{1}{4}, +\infty \right)$

ب) $y = \sqrt{-x^2 + 9x - 5} \xrightarrow{\text{ابتدا به توان ۲ برسانیم}} -x^2 + 9x - 5 \rightarrow \text{ext} \left| \begin{matrix} -\frac{b}{2a} = \frac{9}{2} \\ -\frac{1}{2} \end{matrix} \right| \rightarrow R_f = (-\infty, 4] \rightarrow \sqrt{(-\infty, 4]} = [0, 4] = R_f$

ج) $y = \sqrt{x^2 - 3x^2 + 4x + 1} \rightarrow \text{برای تعریف تابع باید } IR \text{ است} \rightarrow \sqrt{IR} \rightarrow R_f = [0, +\infty)$

د) $y = \log(x^2 - 4x^2 + 7x + 4) \rightarrow R_f = (-\infty, +\infty)$

الف) $y = \frac{3x + 1}{x^2 - 4} \xrightarrow{\text{تابع همگرا نیست}} R_f = IR - \left\{ \frac{a}{c} \right\} \rightarrow R_f = IR - \left\{ \frac{3}{4} \right\}$

ب) $y = \sqrt{\frac{4x + 1}{x - 2}} \xrightarrow{\text{تابع همگرا نیست}} R_f = IR - \left\{ \frac{a}{c} \right\} \rightarrow R_f = IR - \left\{ \frac{1}{2} \right\} \xrightarrow{\text{از اینجا که}} R_f = [0, +\infty) - \left\{ \frac{1}{2} \right\}$

ج) $y = \frac{x^2 + 4}{x^2 + 3} = \frac{x^2 + 3 + 1}{x^2 + 3} = \frac{x^2 + 3}{x^2 + 3} + \frac{1}{x^2 + 3} = 1 + \frac{1}{x^2 + 3} \rightarrow \sqrt{x^2 + 3} = \sqrt{3} \rightarrow \frac{1}{\sqrt{3}} \rightarrow R_f = \left[\frac{1}{\sqrt{3}}, +\infty \right)$

د) $y = x^2 + \frac{1}{x^2 + 4} = x^2 + \frac{1}{x^2 + 4} - 4 \rightarrow \frac{1}{x^2 + 4} = \frac{1}{4} \rightarrow R_f = \left[\frac{1}{4}, +\infty \right)$

الف) $y = \frac{3x^2 + 4}{x^2 - 1} \rightarrow yx^2 - y = 3x^2 + 4 \rightarrow (y - 3)x^2 = 4 + y \rightarrow x^2 = \frac{4 + y}{y - 3} \rightarrow x = \sqrt{\frac{4 + y}{y - 3}}$
 $\rightarrow \frac{-4}{+} \frac{3}{-} \frac{1}{+} \frac{1}{+} \rightarrow R_f = (-\infty, -4] \cup (3, +\infty)$

روشن شود =
 $x^2 \rightarrow \text{مقدار} = 0 \rightarrow x^2 = 0 \rightarrow y = \frac{3(0) + 4}{0 - 1} = -4$
 $x^2 \rightarrow \text{مقدار} = +\infty \rightarrow x^2 = +\infty \rightarrow y = \frac{3(+\infty) + 4}{+\infty - 1} = \frac{3(+\infty)}{(+\infty)} = 3$
 $\rightarrow R_f = (-\infty, -4] \cup (3, +\infty)$

الف) $y = \frac{y \sin x - 1}{\sin x + 3} \rightarrow y \sin x + 3y = y \sin x - 1 \rightarrow (y - y) \sin x = y + 1 \rightarrow \sin x = \frac{y + 1}{y - y}$
 $\rightarrow -1 \leq \sin x \leq 1 \rightarrow -1 \leq \frac{y + 1}{y - y} \leq 1 \rightarrow \frac{y + 1}{y - y} + 1 \geq 0 \rightarrow \frac{y + 1 + y - y}{y - y} = \frac{y + 1}{y - y} \geq 0$
 $\frac{y + 1}{y - y} \leq 1 \rightarrow \frac{y + 1}{y - y} - 1 \leq 0 \rightarrow \frac{y + 1 - y + y}{y - y} = \frac{y + 1}{y - y} \leq 0$
 $\left[-\frac{3}{4}, 2 \right] \cap (-\infty, \frac{1}{4}] \cup (y, +\infty) = \left[-\frac{3}{4}, \frac{1}{4} \right] \rightarrow R_f = \left[-\frac{3}{4}, \frac{1}{4} \right]$

روشن شود =
 $\sin x \rightarrow \text{مقدار} = -1 \rightarrow \sin x = -1 \rightarrow y = \frac{y(-1) - 1}{(-1) + 3} = \frac{-y - 1}{2}$
 $\sin x \rightarrow \text{مقدار} = 1 \rightarrow \sin x = 1 \rightarrow y = \frac{y(1) - 1}{1 + 3} = \frac{y - 1}{4}$
 $\rightarrow R_f = \left[-\frac{3}{4}, \frac{1}{4} \right]$

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$$R_f = (-\infty, -\frac{1}{\mu} \cup +\infty)$$

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$$R_f = [1, 3]$$