

$$\frac{x^2 + x + 2}{x-1} \rightarrow \frac{x^2 + x + 2}{x-1} = y \quad (1)$$

$$y(x-1) = x^2 + x + 2 \rightarrow x^2 + (1-y)x + 2-y = 0 \quad \Delta = b^2 - 4ac$$

$$(1-y)^2 - 4(2-y) \geq 0 \quad y^2 + 1 - 2y - 8 + 4y \geq 0$$

$$y^2 - 4y - 7 \geq 0 \quad \Delta = 16 + 28 = 44 \quad \sqrt{44} = 2\sqrt{11}$$

$$y = \frac{4 \pm 2\sqrt{11}}{2} = 2 \pm \sqrt{11} \quad R_f = \mathbb{R} - (-1, 1)$$

$$(2) y = 2x^2 - 2x + 1 \quad \Delta = b^2 - 4ac = 4 - 8 = -4 < 0$$

$$2\left(\frac{0}{2}\right)^2 - 2\left(\frac{0}{2}\right) + 1 = 1 \quad \frac{20}{1} - \frac{20}{2} + 1 = -10 + 1 = -9 < 0$$

$$\Delta = 4 - 8 = -4 < 0 \quad R_f = [-1, +\infty)$$

$$(3) y = \sqrt{x^2 + 4x} \geq 0 \Rightarrow \frac{-b}{2a} = \frac{-4}{2} = -2 \quad \Delta = 16 - 0 = 16$$

$$\sqrt{(-2) \pm 2} = 0 \Rightarrow (-2, 0) \quad R_f$$

$$(4) y = \sqrt{x^2 + 2x^2 + 2x + 1} \Rightarrow \sqrt{3x^2 + 2x + 1} \quad \Delta = 4 - 12 = -8 < 0$$

$$R_f = \mathbb{R} \quad \text{فردية دالة}$$

$$(5) y = x^3 - 4x^2 + 7x + 2 \geq 0 \Rightarrow \Delta = 16 - 4 = 12 > 0 \quad R_f = (-\infty, +\infty)$$

$$(6) y = \frac{x+1}{2x-4} \Rightarrow x = \frac{-4y-1}{2y-1} \quad \Delta = 16 - 4 = 12 > 0$$

$$R_f = \left[\frac{1}{2}, 2\right] \cup R_f$$



$$1) y = \sqrt{\frac{\epsilon u + 1}{u - r}} \Rightarrow R - \frac{1}{\epsilon}, R - \sqrt{\epsilon}$$

$$\sqrt{R - \sqrt{\epsilon}} : [0, +\infty) - \sqrt{\epsilon} : R \neq$$

$$2) y = \frac{u^2 + \epsilon}{\sqrt{u^2 + r}} \Rightarrow \frac{u^2 + r + 1}{\sqrt{u^2 + r}} \cdot \frac{1}{\sqrt{u^2 + r}} + \sqrt{u^2 + r}$$

$$3) \frac{1 + \sqrt{r}}{\sqrt{r}} \rightarrow \text{کثرين مقدار} \sqrt{r} \quad R_f : \left[\frac{\epsilon}{\sqrt{r}}, +\infty \right)$$

$$4) y = u^2 + \frac{1}{u^2 + \epsilon} \Rightarrow \frac{u^2 + \epsilon + 1}{u^2 + \epsilon} \Rightarrow \frac{\epsilon + 1}{\epsilon} - \frac{\epsilon}{\epsilon} \Rightarrow \frac{1}{\epsilon} - \frac{\epsilon}{\epsilon}$$

$$R_f : \left[\frac{1}{\epsilon}, +\infty \right)$$

$$5) y = \frac{3u^2 + \epsilon}{u^2 - 1} \Rightarrow yu^2 - y = 3u^2 + \epsilon \quad (y - 3)u^2 = \epsilon + y \quad \textcircled{\epsilon}$$

$$u = \pm \sqrt{\frac{\epsilon + y}{y - 3}} \quad R_f : R - (-\epsilon, 3] \quad y - r$$

$$6) y = \frac{3u^2 + \epsilon}{u^2 - 1} \quad u^2 = \square \quad \frac{\epsilon + \epsilon}{-1} = -\epsilon \quad \frac{3(+\infty)}{+\infty} = \frac{3}{1} = \text{max}$$

$$7) R_f : R - (-\epsilon, 3] \quad \text{من مقدار دار تا مقدار منسوب}$$

$$8) \text{من مقدار دار تا مقدار منسوب}$$



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1 $y = \frac{r \sin^2 - 1}{\sin^2 r} \rightarrow y \sin^2 r + r y = r \sin^2 r - 1$ (4)

2 $\sin^2 r (r - y) = r y + 1$ $\sin^2 r = \frac{r y + 1}{r - y}$

3 /

4 $\Rightarrow -1 < \sin^2 r < 1$ $\frac{r-1}{\varepsilon} = \frac{1}{2}$ $\frac{-r-1}{-\varepsilon + r} = \frac{1}{2}$

5

6 $\Rightarrow [-\frac{1}{2}, \frac{1}{2}] = R_f$

7

8 $f(u) = \frac{u^2 - u + 1 + \frac{r}{\varepsilon}}{u^2 - u + 1} = 1 - \frac{r}{\varepsilon(u^2 - u + 1)}$ (4)

9

10 $[0, 1)$ $\Rightarrow$ $\frac{r}{\varepsilon(u^2 - u + 1)}$

12 $y = |2u - 2| - |u - 2|$ (✓)

13 $u < -2 \Rightarrow -2u + 2 + u - 2 = -u$

14 $-u - 1$

15 $u > 2 \Rightarrow 2u - 2 - u + 2 = u$ $R_f = [-2, +\infty)$

16 $y = |u - 2| + |u + 1|$

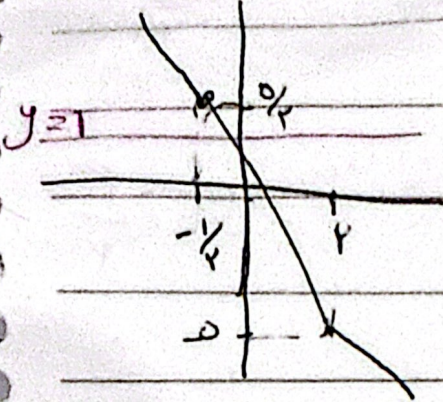
17 $R_f = [+2, +\infty)$

18

$$|u-r| - |ru+1| < 1$$

$$ur \rightarrow u-r-ru-1 \rightarrow -u-r$$

(1)



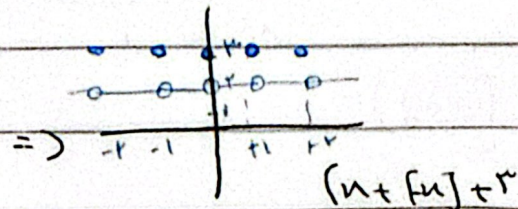
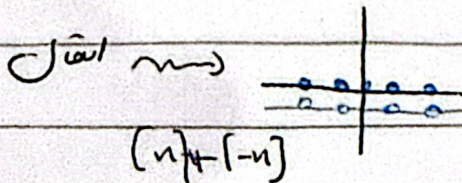
$$x < -\frac{1}{r} - u + r + ru + 1 \Rightarrow u + r$$

$$|u-r| - |ru+1| = 1 \quad \text{Rf. } [-1, +\infty)$$

$$-u+r-ru-1=1$$

$$-ru=1 \quad u = -\frac{1}{r}$$

$$y = [u] + [-u] + r$$



(9)

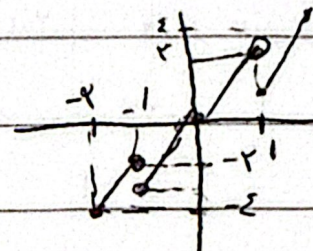
$$\text{Rf. } [r, r]$$

$$y = ru - [u]$$

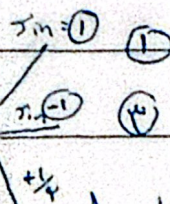
$$-1 < x < 1 \Rightarrow y = ru - 1$$

$$1 < x < 2 \Rightarrow y = ru + 1$$

$$-1 < x < -1 \Rightarrow y = ru + r \quad -1 < x < 0 \Rightarrow y = ru + 1$$



$$y = \sin^2 u - \sin u + 1$$



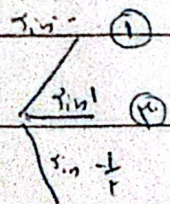
$$\text{Rf. } \left[\frac{1}{2}, \frac{3}{2}\right]$$

(10)

$$\sin u = \frac{1}{2}$$

$$\frac{1}{2} = \frac{1}{r} + 1 \Rightarrow \frac{r}{2}$$

$$y = \sin^2 u + \sin u + 1$$



$$\text{Rf. } [1, 3]$$

$$\sin u = \frac{1}{2}$$

$$\left(\frac{1}{r}\right)^2 + \left(\frac{1}{r}\right) + 1$$

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