آریا توکل

۱) $y = \frac{x^2+x+2}{x-1} \Rightarrow yx - y = x^2 + x + 2 \Rightarrow x^2 + x(1-y) + 2 + y$

$\Delta \geq 0 \Rightarrow 1 + y^2 - 2y - 8 - 4y \geq 0 \Rightarrow y^2 - 6y - 7 \geq 0 \Rightarrow (y-7)(y+1)$; $y = 7$, $y = -1$

	-1		7	
+		−		+

$(-\infty, -1] \cup [7, \infty)$

۲) الف) $y = 2x^2 - 5x + 1$ $\quad \frac{-b}{2a} = \frac{5}{4} \Rightarrow 2(\frac{5}{4})^2 - 5(\frac{5}{4}) + 1 = \frac{-34}{16} = \frac{-17}{8}$

$[-\frac{17}{8}, \infty)$

ب) $y = \sqrt{-x^2 + 6x - 5}$ $\quad \frac{-b}{2a} = \frac{-6}{-2} = 3 \Rightarrow -3^2 + 18 - 5 = 4 \Rightarrow \sqrt{(-\infty, 4]} = [0, 2]$

ج) $\sqrt{x^7 - 3x^2 + 4x + 1}$ $\to R_f = [0, \infty)$

اگر توان بزرگتر فرد باشد برد ما R میشود و چون زیر رادیکال رفته از ۰ تا بینهایت میشود

د) $\log(x^3 - 6x^2 + 7x + 4) \Rightarrow (-\infty, \infty)$; $R_f = R$

۳) الف) $y = \frac{3x+1}{2x-6} \Rightarrow x = \frac{3y+1}{2y-6} \Rightarrow x = \frac{6y+1}{2y-3}$ $\quad R_f = R - \{\frac{3}{2}\}$

ب) $y = \sqrt{\frac{4x+1}{x-2}} \Rightarrow x = \sqrt{\frac{4y+1}{y-2}} \Rightarrow x = \sqrt{\frac{2y+1}{y-4}}$ $\quad R_f = [0, \infty) - \{4\}$

ج) $y = \frac{x^2+4}{\sqrt{x^2+3}} \Rightarrow \sqrt{x^2+3} + \frac{1}{\sqrt{x^2+3}} \Rightarrow \sqrt{x^2+3} = \sqrt{3}_{min}$

$\sqrt{3} + \frac{1}{\sqrt{3}} = \frac{4}{\sqrt{3}}$ $\quad R_f = [\frac{4}{\sqrt{3}}, \infty)$

د) $x^2 + \frac{1}{x^2+4} \Rightarrow x^2 + 4 + \frac{1}{x^2+4} - 4 \Rightarrow [-\frac{1}{4}, \infty)$

(۴)

$y = \frac{3x^2+4}{x^2-1}$ روش اصلی) $yx^2 - y = 3x^2 + 4 \Rightarrow yx^2 - 3x^2 = 4 + y$

$x^2(y-3) = 4+y \rightsquigarrow x^2 = \frac{4+y}{y-3}$ $\quad \frac{-4 \quad 3}{+ \;|\; - \;|\; +}$

$R_f = (-\infty, -4] \cup (3, \infty)$

روش تقریبی) $y = \frac{3(0)+4}{0-1} = -4$ $\quad y = \frac{3(\infty)^2+4}{(\infty)^2-1} = 3$ $\quad f_p = (-\infty, -4] \cup (3, \infty)$

مخرج مساوی صفر نمود.

(۵)

$y = \frac{2\sin x - 1}{\sin x + 3}$ روش اصلی) $y\sin x + 3y = 2\sin x - 1$

$y\sin x - 2\sin x = -1 - 3y \rightarrow \sin x(y-2) = -1-3y$

$\sin x = \frac{-1-3y}{y-2} \rightarrow -1 \leq \frac{-1-3y}{y-2} \leq 1$

$-1 \leq \frac{-3y-1}{y-2} \rightsquigarrow \frac{-2-1}{-1+3} = \frac{-3}{2}$

$\frac{-3y-2}{y-2} \leq 1 \rightsquigarrow \frac{2-1}{1+3} = \frac{1}{4}$ $\Rightarrow \left[-\frac{3}{2}, \frac{1}{4}\right]$

تقریبی) $y = \frac{2\sin - 1}{\sin x + 3}$ $\rightarrow$ شرط -1 $\rightarrow \frac{-2-1}{-1+3} = -\frac{3}{2}$; شرط 1 $\rightarrow \frac{2-1}{1+3} = \frac{1}{4}$ $\Big\} \left[-\frac{3}{2}, \frac{1}{4}\right]$

(۶)

$y = \frac{x^2 - x + \frac{1}{4}}{x^2 - x + 1} \Rightarrow yx^2 - yx + y = x^2 - x + \frac{1}{4}$

$x^2(1-y) + x(y-1) + \frac{1}{4} - y \Rightarrow \Delta \geq 0$

$y^2 + 1 - 2y + (-1+4y)(1-y)$

$y^2 + 1 - 2y - 4y^2 + 4y - 1$

$-3y^2 + 3y \geq 0 \rightarrow 3y(-y+1) \geq 0$ $\quad \frac{- \;|\; + \;|\; -}{0 \quad 1}$

$R_f : [0,1]$

$D_f : \mathbb{R}$ $\quad$ مخرج $x^2 - x + 1$ همیشه مثبت است.

ب)

$y = |2x-2| - |x-3|$

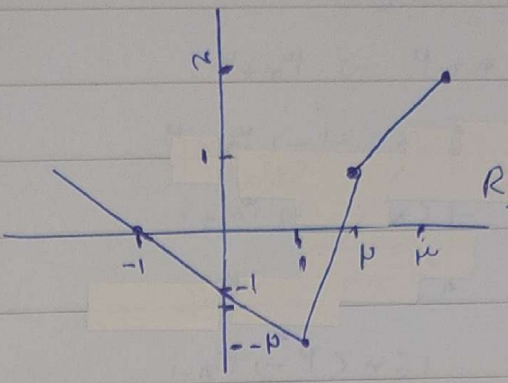
$2x-2 = 0 \Rightarrow x = 1$

$x-3 = 0 \Rightarrow x = 3$

$x < 1$: $y = -2x+2-(-x+3) = -x-1$

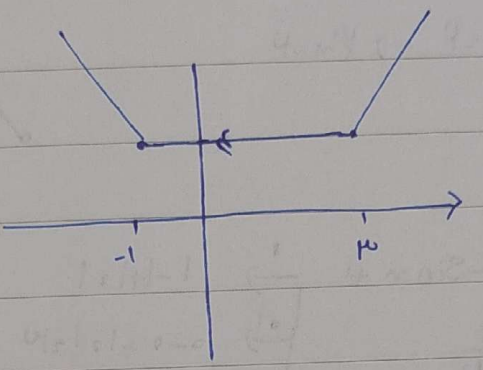
$1 \le x \le 3$: $y = 2x-2-(3-x) = 3x-5$

$x > 3$: $y = 2x-2-(x-3) = x+1$



$R_f = [-2, \infty)$

$y = |x-3| + |x+1|$



ج)

$|x-2| - |2x+2| \le 1$

$x < -1$	$-1 \le x < 2$	$x \ge 2$
$-x+2-2x-2 \le 1$	$x-2-2x-2 \le 1$	$x-2-2x-2 \le 1$
$-3x \le 1$	$-x-4 \le 1$	$-x-4 \le 1$
$x \ge -\frac{1}{3}$ ✗	$x \ge -5$ ✓	$x \ge -5$ ✓

$(-\infty, -5] \cup [-1, \frac{1}{3}] \cup [2, \infty)$

د)

$y = [x] + [-x] + 3$

$x = 3 \Rightarrow y = 3$

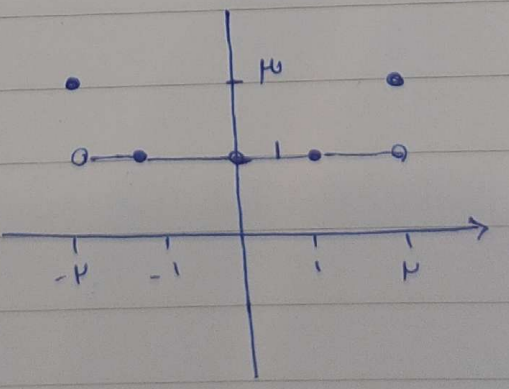
$2 < x < 3 \Rightarrow y = 2$

$1 < x < 2 \Rightarrow y = 3$

$0 < x < 1 \Rightarrow y = 2$

$-1 < x < 0 \Rightarrow y = 3$

$-2 < x < -1 \Rightarrow y = 2$



ب) $y = 3x - [x]$

$x = -2 \rightarrow 3x + 2$

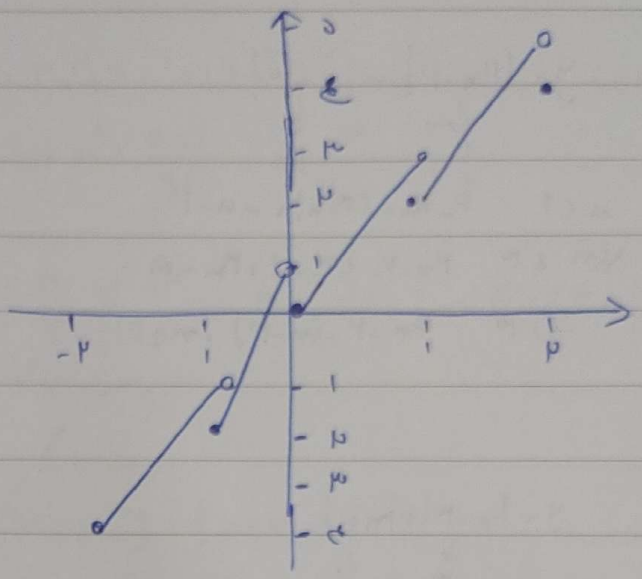
$-2 \leq x < -1 \rightarrow 3x + 2$

$-1 \leq x < 0 \rightarrow 3x + 1$

$0 \leq x < 1 \rightarrow 3x$

$1 \leq x < 2 \rightarrow 3x - 1$

$x = 2 \rightarrow 3x - 2$



(10

$y = \sin^2 x - \sin x + 1$

$-\frac{b}{2a} = \frac{1}{2}$

$x = 1 \rightarrow 1 - 1 + 1 = 1$

$x = -1 \rightarrow 1 + 1 + 1 = 3$

$x = -\frac{b}{2a} \rightarrow \frac{1}{4} - \frac{1}{2} + 1 = \frac{3}{4}$

$\left[\frac{3}{4}, 3\right]$

$y = \sin^4 x + \sin^2 x + 1$

$-\frac{b}{2a} = -\frac{1}{2}$ X

$\sin^2 x$ نمی‌تواند منفی باشد

$x = 1 \rightarrow 1 + 1 + 1 = 3$

$x = 0 \rightarrow 0 + 0 + 1 = 1$

$[1, 3]$