۱

$y=\frac{x^2+x+2}{x-1} \to yx-y=x^2+x+2 \to x^2-x(1-y)+y+2=0 \to x_1,x_2=\frac{-b\pm\sqrt{\Delta}}{2a}$

$\Delta\geq 0 \to (1-y)^2-4(y+2)\geq 0 \to 1+y^2-2y-4y-8\geq 0 \to y^2-6y-7\geq 0 \to (y-7)(y+1)\geq 0$

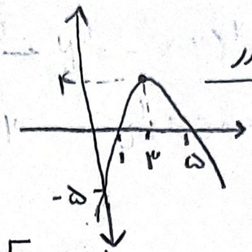
$\begin{array}{c} -1 \quad 7 \\ + \ | \ - \ | \ + \end{array} \to R_f=(-\infty,-1]\cup[7,+\infty)$

۲

الف) $y=2x^2-5x+1 \to R_f=\mathbb{R}$

ب) $y=\sqrt{-x^2+6x-5}$

$\to \frac{-b}{2a}=\frac{-6}{-2}=3 \to y=-9+18-5=4$



جذر $\sqrt{(-\infty,4]}=[0,2]$

ج) $y=\sqrt{x^7-3x^2+4x+1} \to \sqrt{\mathbb{R}} \to R_f=[0,+\infty)$

د) $\log(x^3-6x^2+7x+4) \to (-\infty,+\infty)$

۳

الف) $y=\frac{3x+1}{2x-6} \to \mathbb{R}-\{\frac{3}{2}\}$

ب) $y=\sqrt{\frac{4x+1}{x-2}} \to \sqrt{\mathbb{R}-\{4\}}=[0,+\infty)-\{2\}$

ج) $y=\frac{x^2+4}{\sqrt{x^2+3}} \to \sqrt{x^2+3}+\frac{1}{\sqrt{x^2+3}}=\sqrt{3}+\frac{1}{\sqrt{3}}=\frac{4}{\sqrt{3}}=\frac{4\sqrt{3}}{3}$

د) $y=x^2+\frac{1}{x^2+4}+4-4 \to x^2+4+\frac{1}{x^2+4}-4 \to [-2,+\infty)$

۴

$y=\frac{3x^2+4}{x^2-1} \to yx^2-y=3x^2+4 \to x^2(y-3)=y+4 \to x^2=\frac{y+4}{y-3} \to x=\pm\sqrt{\frac{y+4}{y-3}}$

$\begin{array}{c} -4 \quad 3 \\ + \ | \ - \ | \ + \end{array} \to R_f=(-\infty,-4]\cup(3,+\infty) \to x \to +\infty: \frac{4}{-1}=-4, \quad 0: \frac{4}{-1}=-4$

۵

$y=\frac{2\sin x-1}{\sin x+3} \to y\sin x+3y=2\sin x-1 \to (y-2)\sin x=-3y-1 \to \sin x=\frac{-3y-1}{y-2}$

$\min s=-1 \to y=\frac{-3}{2}$; $\max s=1 \to y=\frac{1}{4}$ $\to R_f=[-\frac{3}{2},\frac{1}{4}]$

$\frac{-3y-1}{y-2}\geq -1 \to \frac{-3y-1+y-2}{y-2}\geq 0 \to \frac{-2y-3}{y-2}\geq 0$

(I) $(-\infty,-\frac{3}{2}]\cup(2,+\infty)$ $\quad \begin{array}{c} -\frac{3}{2} \quad 2 \\ + \ | \ - \ | \ + \end{array}$

$\frac{-3y-1}{y-2}\leq 1 \to \frac{-3y-1-y+2}{y-2}\leq 0 \to \frac{-4y+1}{y-2}\leq 0 \to \begin{array}{c} \frac{1}{4} \quad 2 \\ - \ | \ + \ | \ - \end{array} \to (-\infty,\frac{1}{4}]\cup(2,+\infty)$ (II)

$I\cap II \to R_f=[-\frac{3}{2},\frac{1}{4}]$

۶

$$f(x)=\frac{x^2-x+\frac{1}{4}}{x^2-x+1} \longrightarrow yx^2-yx+y=x^2-x+\frac{1}{4} \longrightarrow x^2(y-1)-x(y-1)+y-\frac{1}{4}=0$$

$$\longrightarrow y_1, y_2=\frac{-b\pm\sqrt{\Delta}}{2a} \longrightarrow (y-1)^2-4\left(y-\frac{1}{4}\right)(y-1)\geq 0 \longrightarrow y^2-2y+1-4y^2+5y-1\geq 0$$

$$\longrightarrow -3y^2+3y\geq 0 \longrightarrow -3y(y-1)\geq 0$$

$D_f=\mathbb{R}$

	0		1	
−	ϕ	+	ϕ	−

$R_f=[0,1]$

۷

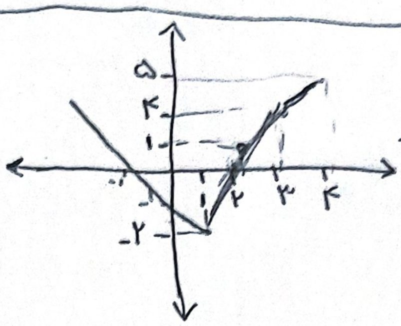
الف) $y=|2x-2|-|x-3|$

x	1		3	
	$-2x+2+x-3 \to -x-1$	$2x-2+x-3 \to 3x-5$		$2x-2-x+3 \to x+1$

x	1	0
y	−2	−1

x	2	3	1
y	1	4	−2

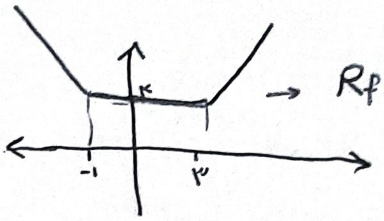
x	3	4
y	4	5



$\longrightarrow R_f=[-2,+\infty)$

ب) $y=|\underset{3}{x-3}|+|\underset{-1}{x+1}|$

x	−1		3	
	$-x+3-x-1 \to -2x+2$	$-x+3+x+1 \to 4$		$x-3+x+1 \to 2x-2$



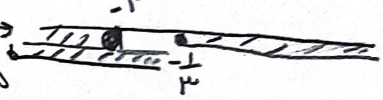
$\longrightarrow R_f=[4,+\infty)$

۸

$|x-2|-|2x+2|\leq 1$

−1		2	
$-x+2+2x+2\leq 1$, $x\leq -3$ (I)	$-x+2-2x-2\leq 1$, $-3x\leq 1$, $x\geq -\frac{1}{3}$ (II)		$x-2-2x-2\leq 1$, $-x\leq 5 \to x\geq -5$ (III)

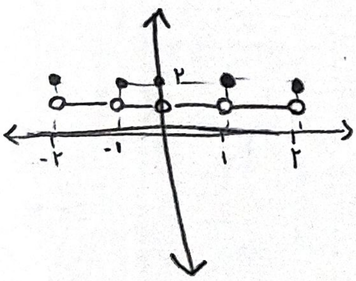
$I\cap II\cap III \longrightarrow$



$[-5,-3]$ جواب نامعادله

۹

الف) $y=[x]+[-x]+3$ — $x\in\mathbb{Z} \to 3$ ، $x\notin\mathbb{Z} \to 2$



ب) $y=3x-[x]$

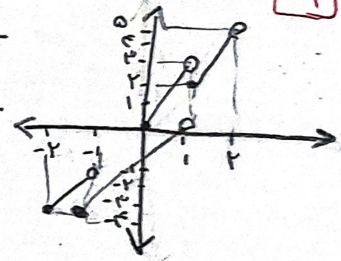
$-2\leq x<-1 \xrightarrow{[x]=-2} 3x+2$ — x: −2, −1 ; y: −4, −1

$-1\leq x<0 \xrightarrow{[x]=-1} 3x+1$ — x: −1, 0 ; y: −2, 1

$0\leq x<1 \xrightarrow{[x]=0} 3x$ — x: 0, $\frac{1}{3}$; y: 0, 1

$1\leq x<2 \xrightarrow{[x]=1} 3x-1$ — x: 1, 2 ; y: 2, 5

$x=2 \longrightarrow y=4$



۱۰

الف) $y=\sin^2x-\sin x+1$

- $1 \to 1$
- $-1 \to 3$
- $-\frac{b}{2a} \to \left(\frac{1}{2}\right)^2-\frac{2}{4}+\frac{4}{4}=\frac{3}{4}$

$\longrightarrow R_f=\left[\frac{3}{4},3\right]$

ب) $y=\sin^4x+\sin^2x+1$

- $1 \to 3$
- $0 \to 1$
- $-\frac{b}{2a} \to \left(-\frac{1}{2}\right)^4+\left(-\frac{1}{2}\right)^2+1=\frac{1}{16}+\frac{4}{16}+\frac{16}{16}=\frac{21}{16}=1\frac{5}{16}$

$\longrightarrow R_f=[1,3]$