1-

$y=\frac{x^2+x+2}{x-1}$ $\quad y=\frac{\left(x+\frac{1}{2}\right)^2+\frac{7}{4}}{x-1}$ $\quad (\to x^2+x+\frac{1}{4})$ $\quad \left(x+\frac{1}{2}\right)^2+\frac{7}{4}=y(x-1)$

$\left(x+\frac{1}{2}\right)^2+\frac{7}{4}-y(x-1)=0$

$\left(x+\frac{1}{2}\right)^2-y\left(x+\frac{1}{2}-\frac{3}{2}\right)+\frac{7}{4}=0 \Rightarrow \left(x+\frac{1}{2}\right)^2-y\left(x+\frac{1}{2}\right)+\frac{3}{2}y+\frac{7}{4}=0$

$b^2-4ac=y^2-4\left(\frac{6y+7}{4}\right) \Rightarrow y^2-6y-7\geq 0 \Rightarrow (y-7)(y+1)\geq 0$

$R_f=(-\infty,-1]\cup[7,+\infty)$

الف) $2x^2-5x+1=y$ $\quad a>0$ $\quad$ رأس $y=-\frac{\Delta}{4a}=\frac{-b^2+4ac}{4a}=\frac{-25+8}{8}=-\frac{17}{8}$ $\quad$ -2

$\Rightarrow R_f=\left[-\frac{17}{8},+\infty\right)$

ب) $\sqrt{-x^2+6x-5}=\sqrt{-(x^2-6x+5)}=\sqrt{-(x-3)^2+4}=y \Rightarrow R_f=[0,2]$

$-x^2+6x-9$

$\to -\frac{b}{2a}=-\frac{6}{-2}=3$

$\sqrt{-9+18-5}=\sqrt{4}=2$

ج) $y=\sqrt{x^7-3x^2+4x+1} \Rightarrow R_f=[0,+\infty)$

د) $y=\log(x^3-6x^2+7x+4) \to >0 \quad 10^y>0 \Rightarrow$

الف) $y=\frac{3x+1}{2x-6}$ $\quad 2xy-6y=3x+1$ $\quad 2xy-3x=1+6y$ $\quad$ -3

$x(2y-3)=1+6y \Rightarrow x=\frac{1+6y}{2y-3}$ $\quad R_f=\mathbb{R}-\left\{\frac{3}{2}\right\}$

ب) $y=\sqrt{\frac{4x+1}{x-2}} \to$ $\begin{array}{c|c|c} \multicolumn{1}{r}{-\frac{1}{4}} & \multicolumn{1}{r}{2} & \\ + & - & + \end{array}$ $\quad D_f=\left(-\infty,-\frac{1}{4}\right]\cup(2,+\infty)$

$\Rightarrow R_f=[0,+\infty)$

ج) $y = \dfrac{x^2+4}{\sqrt{x^2+3}}$ $\xrightarrow{}$ صعودی , $\sqrt{x^2+3} \xrightarrow{}$ صعودی $\Rightarrow x=0 \Rightarrow \dfrac{4}{\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{4\sqrt{3}}{3} \Rightarrow R_f = \left[\dfrac{4\sqrt{3}}{3}, +\infty\right)$

د) $y = x^2 + \dfrac{1}{x^2+4}$ $\left(x^2+4+\dfrac{1}{x^2+4}\right) - 4 = y$

$\xrightarrow{} x=0 \Rightarrow 4+\dfrac{1}{4} = \dfrac{17}{4} \Rightarrow R_f = \left[\dfrac{17}{4}, +\infty\right)$

$y = \dfrac{3x^2+4}{x^2-1}$ $yx^2 - y = 3x^2+4$ $yx^2 - 3x^2 = 4+y$ $x^2(y-3) = 4+y$ _۴

$x^2 = \dfrac{4+y}{y-3}$ $\dfrac{4+y}{y-3} \geq 0$ $y \neq 3$ $\begin{array}{c|c|c} & -4 \quad 3 & \\ + & - & + \end{array}$

$\Rightarrow R_f = (-\infty, -4] \cup (3, +\infty)$

جواب نهایی $\Rightarrow (-\infty, -4] \cup (3, +\infty)$ $x=0 \Rightarrow \dfrac{+4}{-1} = -4$ $\dfrac{a}{c} = 3$

$y = \dfrac{2\sin x - 1}{\sin x + 3}$ $y\sin x + 3y = 2\sin x - 1$ _۵

$2\sin x - y\sin x = -1-3y \Rightarrow \sin x(2-y) = -(3y+1)$

$\Rightarrow \sin x = \dfrac{3y+1}{y-2}$ $-1 \leq \dfrac{3y+1}{y-2} \leq 1$

$\Rightarrow \dfrac{3y+1-y+2}{y-2} \leq 0 \Rightarrow \dfrac{2y+3}{y-2} \leq 0 \Rightarrow \left[-\dfrac{3}{2}, 2\right)$

$\dfrac{3y+1+y-2}{y-2} \geq 0 \Rightarrow \dfrac{4y-1}{y-2} \geq 0 \Rightarrow \left(-\infty, \dfrac{1}{4}\right] \cup (2, +\infty) \Rightarrow R_f = \left[-\dfrac{3}{2}, \dfrac{1}{4}\right]$

$\sin x = 1 \Rightarrow \dfrac{2-1}{4} = \dfrac{1}{4}$, $\dfrac{-2-1}{-1+3} = -\dfrac{3}{2}$ جواب نهایی $\xrightarrow{}$ $\left[-\dfrac{3}{2}, \dfrac{1}{4}\right]$

$f(x) = \dfrac{\left(x-\frac{1}{2}\right)^2}{\left(x-\frac{1}{2}\right)^2 + \frac{3}{4}}$ $D_f = \mathbb{R}$ $R_f = [0, 1)$ ۶

$\xrightarrow{}$ همیشه مثبت

۷

الف) $y = |2x-2| - |x-3|$

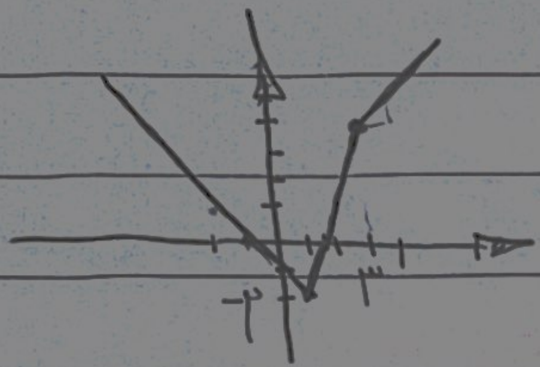
$1 \quad 3$

$2x-2-x+3 = x+1$

$2x-2+x-3 = 3x-5 = 0$

$-2x+2+x-3 = -x-1 = 0$

$2x-2 = x-3 \Rightarrow x = -1$



$\Rightarrow R_f = [-2, +\infty)$

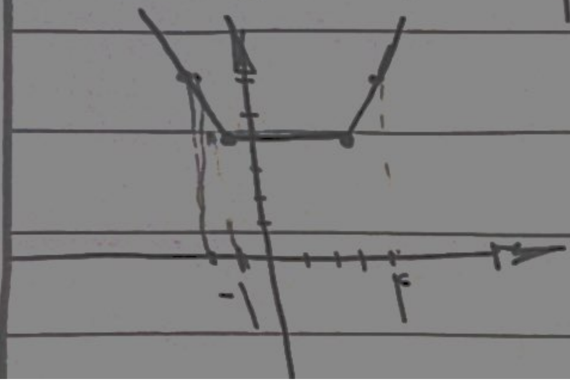
ب) $y = |x-3| + |x+1|$

$-1 \quad 3$

$-x+3-x-1 = -2x+2$

$x-3+x+1 = 2x-2$

$-x+3+x+1 = 4$



$R_f = [4, +\infty)$

8 -

$$|x-2|-|2x+2|\leqslant 1$$

$x<-1$	-1	$-1\le x\le 2$	2	$x>2$
$-x+2+2x+2$	3	$-x+2$	-6	$x-2-2x-2$
$=x+4$		$-2x-2$		$=-x-4$
$x+4\leqslant 1$		$=-3x\leqslant 1$		$-x-4\leqslant 1$
$x\leqslant -3$		$x\geqslant -1/3$		$x\geqslant -5 \to [-5,2]$
$\to (-\infty,-3]$		$\to [-1/3,2]$		

$$\Rightarrow (-\infty,-3]\cup[-5,2]$$

$5-6 \quad -3 \quad -6 \quad -4 \quad -8$

$-2 \quad +6 \quad 6$

۶ -

الف) $y=[x]+[3-x]$

$x=0 \to 3$

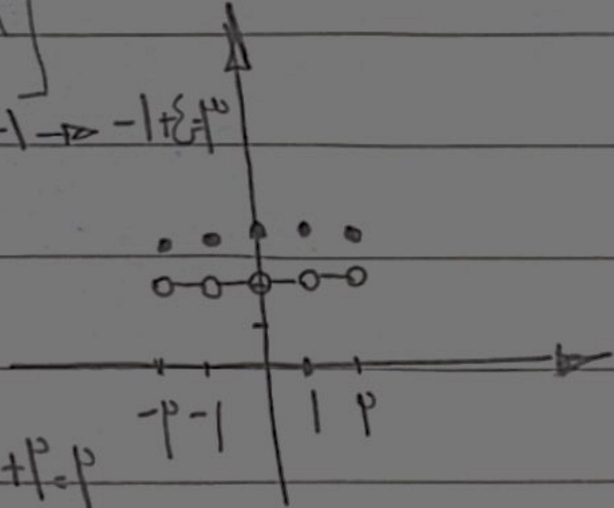
$x=-1 \to -1+4=3$

$x=1 \to 1+2=3$

$x=2 \to 2+1=3$

$x=\frac{1}{2} \to [\frac{1}{2}]+[\frac{5}{2}]=0+2=2$

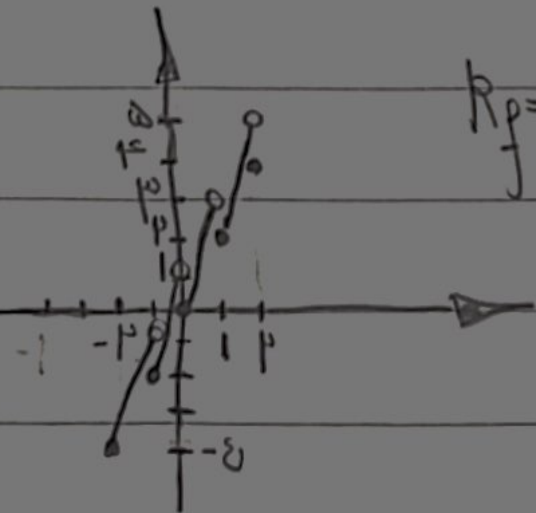
$x=\frac{1}{4} \to [\frac{1}{4}]+[\frac{11}{4}]=0+2=2$



$x=\frac{5}{6} \to 0+[\frac{18-5}{6}]=2$

$R_f=\{2,3\}$

ب) $y=3x-[x]$



$R_f=[-4,5)$

$x=0 \to 0$

$x=-2 \to -6+2=-4$

$x=-1 \to -3+1=-2$

الف) $y = \sin^2 x - \sin x + 1$ $\qquad 1 \geq \sin x \geq -1 \qquad -\frac{b}{2a} = +\frac{1}{2}$ $\qquad$ -۱۰

$y = \frac{1}{4} - \frac{1}{2} + 1 = \frac{1}{4} - \frac{2}{4} + \frac{4}{4} = \frac{3}{4} \Rightarrow R_f = [-1, +1]$

ب) $y = \sin^4 x + \sin^2 x + 1$ $\qquad 1 \geq \sin^2 x \geq 0 \qquad -\frac{b}{2a} = -\frac{1}{2} \rightarrow$ غ‌ق‌ق

$R_f = [0, 1]$