۱۴

۲

$y=\dfrac{x^2+x+2}{x-1}$ $\quad y=\dfrac{\left(x+\frac{1}{2}\right)^2+\frac{7}{4}}{x-1}$ $\quad \left(x+\frac{1}{2}\right)^2+\frac{7}{4}=y(x-1)$

$\left(x+\frac{1}{2}\right)^2+\frac{7}{4}-y(x-1)=0$

$\left(x+\frac{1}{2}\right)^2-y\left(x+\frac{1}{2}-\frac{3}{2}\right)+\frac{7}{4}=0 \Rightarrow \left(x+\frac{1}{2}\right)^2-y\left(x+\frac{1}{2}\right)+\frac{3}{2}y+\frac{7}{4}=0$

$b^2-4ac=y^2-4\left(\frac{6y+7}{4}\right) \Rightarrow y^2-6y-7\geq 0 \Rightarrow (y-7)(y+1)\geq 0$

$R_f=(-\infty,-1]\cup[7,+\infty)$

۱٫۵

۳ـ

الف) $2x^2-5x+1=y$ $\quad a>0$ $\quad$ رأس $y=-\dfrac{\Delta}{4a}=\dfrac{-b^2+4ac}{4a}=\dfrac{-25+8}{8}=-\dfrac{17}{8}$

$\Rightarrow R_f=\left[-\frac{17}{8},+\infty\right)$

ب) $\sqrt{-x^2+6x-5}=\sqrt{-(x^2-6x+5)}=\sqrt{-(x-3)^2+4}=y \Rightarrow R_f=[0,2]$

$-\dfrac{b}{2a}=\dfrac{-6}{-2}=3$ $\quad -x^2+6x-9$

$\sqrt{-9+18-5}=\sqrt{4}=2$

ج) $y=\sqrt{x^7-3x^3+4x+1} \Rightarrow R_f=[0,+\infty)$

د) $y=\log(x^3-6x^2+7x+4) \to >0 \quad 10^y>0 \Rightarrow$? $\longrightarrow R_f=\mathbb{R}$

۱٫۷۵

۴ـ

الف) $y=\dfrac{3x+1}{2x-6}$ $\quad 2xy-6y=3x+1$ $\quad 2xy-3x=1+6y$

$x(2y-3)=1+6y \Rightarrow x=\dfrac{1+6y}{2y-3}$ $\quad R_f=\mathbb{R}-\left\{\frac{3}{2}\right\}$

ب) $y=\sqrt{\dfrac{4x+1}{x-2}}$ $\quad -\frac{1}{4}\ \ 2$: $+\ |\ -\ |\ +$ $\quad D_f=\left(-\infty,-\frac{1}{4}\right]\cup(2,+\infty)$

$\Rightarrow R_f=[0,+\infty)$

۰٫۲۵ $\quad R_f=\mathbb{R}-\{4\}$ $\quad \longrightarrow R_f=[0,+\infty)-\{2\}$

ج) $y = \frac{x^2+4}{\sqrt{x^2+3}}$ مینیمم

$\Rightarrow x=0 \Rightarrow \frac{4}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{4\sqrt{3}}{3} \Rightarrow R_f = [\frac{4\sqrt{3}}{3}, +\infty)$

د) $y = x^2 + \frac{1}{x^2+4}$ $\left(x^2+4+\frac{1}{x^2+4}\right) - 4 = y$

$x = 0 \Rightarrow 4 + \frac{1}{4} = \frac{17}{4} \Rightarrow R_f = [\frac{1}{4}, +\infty)$

۴ -۴

$y = \frac{3x^2+4}{x^2-1}$ $yx^2 - y = 3x^2+4$ $yx^2 - 3x^2 = 4+y$ $x^2(y-3) = 4+y$

$x^2 = \frac{4+y}{y-3}$ $\frac{4+y}{y-3} \geq 0$ $y \neq 3$

	-4		3	
+		-		+

$\Rightarrow R_f = (-\infty, -4] \cup (3, +\infty)$

تو نمودار نشون میده $\Rightarrow (-\infty, -4] \cup (3, +\infty)$ $x=0 \Rightarrow \frac{+4}{-1} = -4$ $\frac{a}{c} = 3$

۲ -۵

$y = \frac{2\sin x - 1}{\sin x + 3}$ $y\sin x + 3y = 2\sin x - 1$

$2\sin x - y\sin x = -1 - 3y \Rightarrow \sin x(2-y) = -(3y+1)$

$\Rightarrow \sin x = \frac{3y+1}{y-2}$ $-1 \leq \frac{3y+1}{y-2} \leq 1$

$\Rightarrow \frac{3y+1-y+2}{y-2} \leq 0 \Rightarrow \frac{2y+3}{y-2} \leq 0 \Rightarrow [-\frac{3}{2}, 2)$

$\frac{3y+1+y-2}{y-2} \geq 0 \Rightarrow \frac{4y-1}{y-2} \geq 0 \Rightarrow (-\infty, \frac{1}{4}] \cup (2, +\infty) \Rightarrow R_f = [-\frac{3}{2}, \frac{1}{4}]$

$\sin x = 1 \Rightarrow \frac{2-1}{4} = \frac{1}{4}$, $\frac{-2-1}{-1+3} = -\frac{3}{2}$ جواب صحیح $[-\frac{3}{2}, \frac{1}{4}]$

۱ ۹

$f(x) = \frac{(x-\frac{1}{2})^2}{(x-\frac{1}{2})^2 + \frac{3}{4}}$ $D_f = \mathbb{R}$ $R_f = [0, 1)$?

مخرج صفر نمیشه

کامل‌تر بنویسید

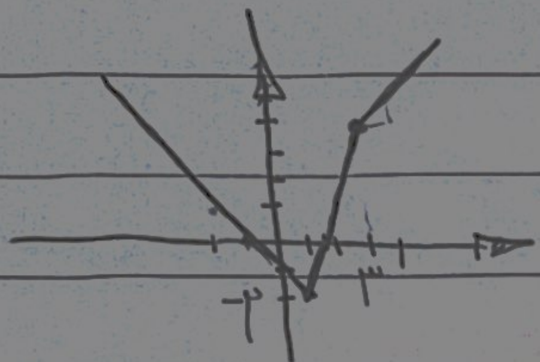
الف) $y = |2x-2| - |x-3|$

$x > 3: \quad 2x-2-x+3 = x+1$

$1 \le x \le 3: \quad 2x-2+x-3 = 3x-5 = 0$

$x < 1: \quad -2x+2+x-3 = -x-1 = 0$

$2x-2 = x-3 \Rightarrow x = -1$



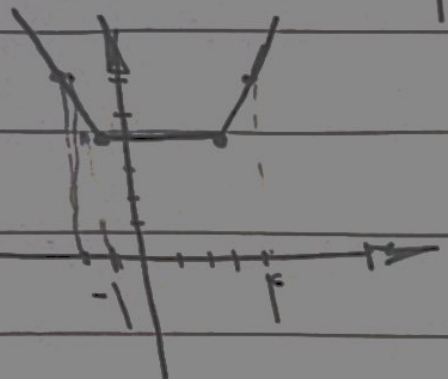
$\Rightarrow R_f = [-2, +\infty)$

ب) $y = |x-3| + |x+1|$

$x > 3: \quad x-3+x+1 = 2x-2$

$x < -1: \quad -x+3-x-1 = -2x+2$

$-1 \le x \le 3: \quad -x+3+x+1 = 4$



$R_f = [4, +\infty)$

۲

۱,۷۵

۸-

$|x-۲| - |۲x+۲| \leq ۱$

$x < -۱$	$-۱ \leq x \leq ۲$	$x > ۲$
$-x+۲+۲x+۲$	$-x+۲$	$x-۲-۲x-۲$
$= x+۴$	$-۲x-۲$	$= -x-۴$
$x+۴ \leq ۱$	$= -۳x \leq ۱$	$-x-۴ \leq ۱$
$x \leq -۳$	$x \geq -\frac{۱}{۳}$	$x \geq -۵ \rightarrow [-۵, ۲]$
$\rightarrow (-\infty, -۳]$	$\rightarrow [-\frac{۱}{۳}, ۲]$	

$\Rightarrow (-\infty, -۳] \cup [-۵, ۲]$

$R_f = (-\infty, -۳] \cup [-\frac{۱}{۳}, +\infty)$

۲

-۹

الف) $y=[x]+[۳-x]$

$x=۰ \rightarrow ۳$

$x=۱ \rightarrow ۱+۲=۳$

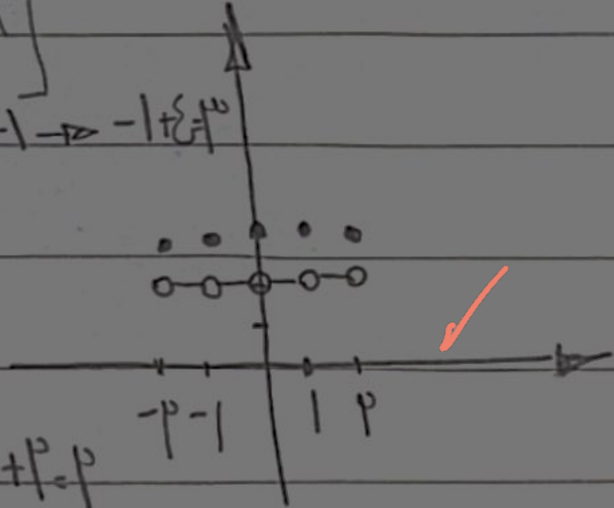
$x=۲ \rightarrow ۲+۱=۳$

$x=۱/۲ \rightarrow [۱/۲]+[۵/۲]=۰+۲=۲$

$x=۱/۴ \rightarrow [۱/۴]+[۱۱/۴]=۰+۲=۲$

$x=-۱ \rightarrow -۱+۴=۳$

$x=۵/۶ \rightarrow ۰+[۱۸-\cancel{۵}/۶]=۲$



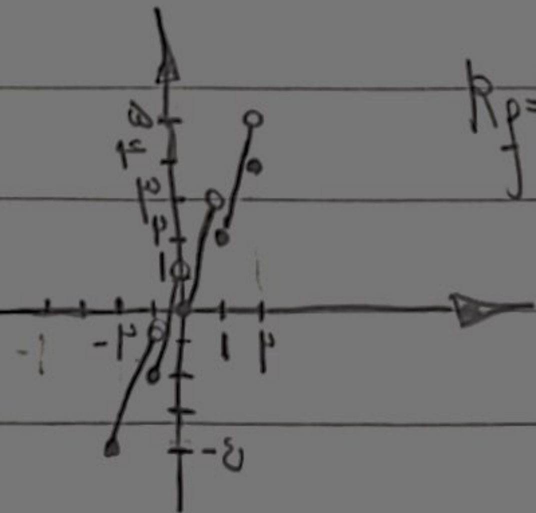
$R_f=\{۲,۳\}$

ب) $y=۳x-[x]$

$x=۰ \rightarrow ۰$

$x=-۲ \rightarrow -۶+۲=-۴$

$x=-۱ \rightarrow -۳+۱=-۲$



$R_f=[-۴,۵)$

الف) $y = \sin^2 x - \sin x + 1 \qquad 1 \geq \sin x \geq -1 \qquad -\frac{b}{2a} = +\frac{1}{2}$

$y = \frac{1}{4} - \frac{1}{2} + 1 = \frac{1}{4} - \frac{2}{4} + \frac{4}{4} = \frac{3}{4} \Rightarrow R_f = [-1, +1]$

$\sin x = -1 \rightarrow 3$

$\sin x = 1 = 1$

-1

0

$\sin x = \frac{-b}{2a} = \frac{1}{2} \rightarrow \frac{3}{4}$

ب) $y = \sin^4 x + \sin^2 x + 1 \qquad 1 \geq \sin^2 x \geq 0 \qquad -\frac{b}{2a} = -\frac{1}{2} \rightarrow$ غق $\quad R_f = [\frac{3}{4}, 3]$

$R_f = [0, 1]$

$\sin^2 x = 0 \rightarrow 1$

$\sin^2 x = 1 \rightarrow 3$

$\} \quad R_f = [1, 3]$