۱

$f(0)=0 \Rightarrow 1+b=0 \Rightarrow b=-1 \quad -6(2\cos(2x)\times\cos^3 x)+4\cos(x)\sin(-1))$

$f'(0)=0 \quad f''(0)=2$

$\cancel{-\sin(2\times 0)}\ 2\cos^3(0)+2ax$

$-6\sin(2x)\cos^3 2x+2ax$

$\frac{-6(2\cos(x)\cos^3(2x))}{2}+\frac{4\cos(2x)\sin 2x)}{0}+2a$

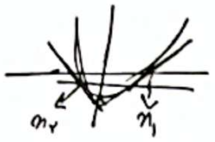
$\boxed{a+b=6}$

$-12+2a=2 \Rightarrow a=7$

۲

$f(x)=x^2-1 \Rightarrow \underbrace{y=k}_{\text{خط}} \ /\ x^2-1=k \Rightarrow x_{1,2}=\pm\sqrt{k+1}$

$f'(x)=2x$



$2x_1\times 2x_2=-1 \Rightarrow x_1\times x_2=-\frac{1}{4}$

$\Rightarrow k+1=\frac{1}{4} \Rightarrow k=-\frac{3}{4} \quad -\frac{3}{4}-\frac{3}{4}=\boxed{-\frac{6}{4}}$

۳

$A(-\frac{1}{3},-12)\ B(\frac{2}{3},6) \to m_{AB}=\frac{18}{3}=6 \quad y_{\text{مماس}}=6x-9 \quad f(x)=\frac{a}{3x-1}$

$6x-9=\frac{x}{3x-1} \Rightarrow 18x^2-6x-18x+9=a \Rightarrow 18x^2-24x+9=a \ \}\ 4x^2-1=-a$

$f'(x)=6 \Rightarrow \frac{-3a}{(3x-1)^2}=6 \Rightarrow 24x^2-24x+6=-2$

جایگذاری $\quad -12x^2+24x-9=4x^2-1 \Rightarrow 16x^2-24x+8=0 \begin{cases} x=1 \Rightarrow a=-3 \\ x=\frac{1}{2} \Rightarrow a=0 \text{ غ ق ق} \end{cases}$

$f(0)=-\frac{3}{9}=\boxed{-\frac{1}{3}}$

۴

$y=\frac{x+a}{ax+1} \Rightarrow y'=\frac{ax+1-a(x+a)}{(ax+1)^2}=\frac{1-a^2}{(ax+1)^2}=2\ (x+1)$

$\frac{1-a^2}{a^2+2a+1}=2 \Rightarrow 2a^2+4a+2=1-a^2 \to 3a^2+4a+1=0 \begin{cases} a=-1 \text{ غ ق ق} \\ a=-\frac{1}{3} \end{cases}$

$y=\frac{x-\frac{1}{3}}{-\frac{1}{3}x+1} \quad f(1)=1 \to 2+b=1 \quad b=-1 \quad a-b=-\frac{1}{3}+1=\boxed{\frac{2}{3}}$

۵

$f(x)=g(x) \Rightarrow \sin x+\frac{1}{2}\cos x=\frac{3}{2}\sin x \Rightarrow \sin x=\cos x \quad [0,\pi]\ x=\frac{\pi}{4}$

$f(\frac{\pi}{4})=\frac{2\sqrt{2}+\sqrt{2}}{4}=\frac{3\sqrt{2}}{4} \quad f'(x)=\cos x-\frac{1}{2}\sin x \to f'(\frac{\pi}{4})=\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{4}=\frac{\sqrt{2}}{4}$

$y=\frac{\sqrt{2}}{4}x+\frac{3\sqrt{2}}{4}-\frac{\sqrt{2}\pi}{16} \quad (y=0) \Rightarrow x=\frac{\pi-12}{4}$

$f'=0 \Rightarrow A\,\Big|^{-1}_{8}$ $\quad B(2,-19)$

$\Rightarrow$ دونقطه $\quad m_{AB}=-9$

موازی اند $\Rightarrow m_{\text{مماس}} = m_{AB} = -9 \Rightarrow f'(-9)=$ $x_C=-1$, $x_D=2$ ۶

$6x^2-6x-12=-9$

$\Rightarrow 6x^2-6x-3=0$

$y=kx^3+(k+1)x^2 \longrightarrow$ نقطه عطف $=\dfrac{-(k+1)}{3k}<0 \Rightarrow \dfrac{k+1}{3k}>0$ $\quad$ $-1 \quad 0$: $+ \; - \; +$ $\quad$ ① ج

$f\left(-\dfrac{(k+1)}{3k}\right)>0 \Rightarrow k\left(\dfrac{-(k+1)}{3k}\right)^3+(k+1)\left(\dfrac{-(k+1)}{3k}\right)^2>0 \Rightarrow \dfrac{2(k+1)^3}{27k^2}>0$ ۷

$2(k+1)^3>0 \Rightarrow (k+1)^3>0 \quad k+1>0 \Rightarrow k>-1$ ② ج

$① \cap ② = \varnothing$ — جمع مقادیر k از

$y=x^3+ax^2+bx-1 \xrightarrow{(-1,-4)} -2+a-b=-4 \Rightarrow \begin{cases} 2a-2b=-4 \\ -2a+b=-4 \end{cases}$

$y'=3x^2+2ax+b \xrightarrow{(-1,-4)} 3-2a+b=-4$

$-b=-8 \Rightarrow b=8,\ a=6$ ۸

$\dfrac{a}{b}=\dfrac{6}{8}=\boxed{\dfrac{3}{4}}$

$f(x)=x^4-6x^2+5 \to f'(x)=4x^3-12x \to f''(x)=12x^2-12=0 \Rightarrow x=1,-1$

$\Rightarrow C(-1,0),\ D(1,0)$ $\quad 4x(x^2-3)$: $0,\ \sqrt{3},\ -\sqrt{3}$ $\quad$ $-\sqrt{3}\ \ 0\ \ \sqrt{3}$: $-\ +\ -\ +$ min max min

$\overline{AB}$ و $\overline{CD}$ هر دو خطوط موازی محور x هستند و زاویه بین آن صفر است. ۱۰

$f(0)=4 \Rightarrow c=4$ $\qquad f\left(-\dfrac{2a}{3}\right)=0 \longrightarrow \left(-\dfrac{2a}{3}\right)^3+a\left(-\dfrac{2a}{3}\right)^2+4=0$

$f'=0 \Rightarrow 3x^2+2ax+b \xrightarrow{x=0} 0 \Rightarrow b=0$

Min نسبی $\xrightarrow{f'(x)=0} 3x^2+2ax=0 \Rightarrow x=0,\ -\dfrac{2a}{3}$ x_{min} $\qquad \Rightarrow a=-3$ ۹

$\Rightarrow \boxed{x_{min}=2}$