Subject. Day. Month. Year.

۱۶٫۲۵ — روزبین فرزیان

① $f(x)=1-\frac{a}{x}$ $[1\ 4]$

$1,1-a$ $\quad 4, 1-\frac{a}{4}$ $\quad m=\frac{1-\frac{a}{4}-1+a}{3}=\frac{3a}{12}=\frac{a}{4}$ (۱٫۷۵)

$f'(x)=\frac{a}{x^2}=\frac{a}{4}\rightarrow x=\pm\sqrt{4}$ $\begin{cases}x=-\sqrt{4} & ✗\\ x=\sqrt{4} & ✓\end{cases}$

② $y=2ax^2-5x+18a$ $\quad y=x$ $\quad 4ax-5=1$ $\quad 4ax=6$ $\quad ax=\frac{3}{2}$ (۱٫۵)

$y'=4ax-5$ $\quad y'=1$

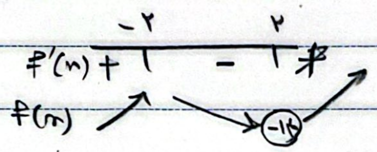
$y=2(ax)x-5x+18a=x$ $\quad 3x-5x+18a=x$ $\quad 3x=18a$ $\quad x=6a$

$6a^2=\frac{3}{2}$ $\quad a^2=\frac{1}{4}\rightarrow a=\pm\frac{1}{2}$ $\quad x=\pm 3$ $\quad x=-3,\ a=-\frac{1}{2}$

$ax^2-6x+9a=0 \xrightarrow{\Delta=0} 9-4(a)(9a)=0 \rightarrow 9-36a^2=0 \rightarrow a^2=\frac{1}{4}\rightarrow a=\pm\frac{1}{2}\rightarrow a=\frac{-1}{2}$

$a=\frac{1}{2}\rightarrow$ غیرقابل قبول $\rightarrow x^2-6x+9=(x-3)^2=0\rightarrow$ ریشه مضاعف ← غلط

③ $y=x^3-12x+2\rightarrow y'=3x^2-12=0\rightarrow x=\pm 2$ (۲)



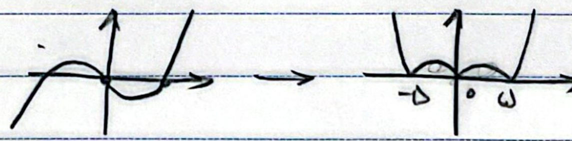
$y=-14$ ✓

④ $y=x^3+ax^2-2bx-4\rightarrow y'=3x^2+2ax-2b \xrightarrow[\text{مستقیم تابع را قطع می‌کند}]{-2,\ 0} -2b=0\rightarrow b=0$

$y'=12-4a-0=0\rightarrow \boxed{a=3}\rightarrow y=x^3+3x^2-4$ $\quad f(0)=-4$ $\quad f(-2)=0$ (۲)

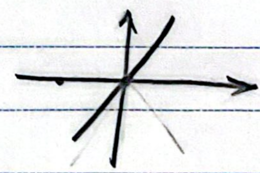
$\sqrt{4+16}=\boxed{2\sqrt{5}}$ ✓

⑤ $f(x)\begin{cases}x^2-5x & x\geq 0\\ x^2+5x & x\leq 0\end{cases}$ $\quad x(x-5)$

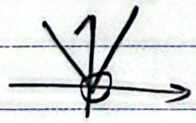


$(x)\max=2$ $\quad (x)\min=\pm 4$ $\quad \frac{n}{m}=1/5$ (۲) ✓

⑥ $f(x)=x(|x|+3)$ $\begin{cases}x^2+3x & x\geq 0\\ -x^2+3x & x\leq 0\end{cases}$ $\quad x(x+3)$ $\quad x(3-x)$



$f(|x|)=$



$x=0$ نقطه بحرانی (۲) ✓

⑦ (۱) $f(x)=\sqrt[3]{x^2}\,|x-a| \xrightarrow{x\leq a} \sqrt[3]{x^2}(x-a)=y \rightarrow y=(x^{\frac{2}{3}})(a-x)=y$

$y'=(\frac{2}{3}x^{-\frac{1}{3}})(a-x)+(-1)(x^{\frac{2}{3}})=0$

[illegible]

$\frac{2}{3}ax^{-\frac{1}{3}}-\frac{2}{3}x^{\frac{2}{3}}-x^{\frac{2}{3}}=0$

$\frac{1}{3}x^{-\frac{1}{3}}(2a-2x-x)=0$ $\quad 2a=3x$ $\quad a=1/5x$

$(x^{\frac{2}{3}})(\frac{1}{2}x)=\frac{3}{2}$ $\quad \frac{1}{2}x^{\frac{5}{3}}=\frac{3}{2}$ $\quad \boxed{x=\sqrt[5]{3^3}}$

$a=\frac{3}{2}\times\sqrt[5]{3^3}=\frac{3\sqrt[5]{3^3}}{2}$

$f(x_{max})=1/5\rightarrow f(\frac{2}{5}a)=\frac{3}{2}\rightarrow \sqrt[3]{\frac{4}{25}a^2}(\frac{2}{5}a-a)=\frac{3}{2}\rightarrow a\times\sqrt[3]{\frac{4}{25}a^2}=\frac{5}{2}$

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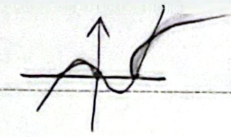
$\xrightarrow{\text{توان ۳}} a^3\times\frac{4}{25}a^2=\frac{125}{8}\rightarrow a^5=\frac{125}{8}\times\frac{25}{4}=(\frac{5}{2})^5\rightarrow a=\frac{5}{2}=2/5$

⑧ $f(x)=\sqrt{x|x|-x} \longrightarrow h(x)=x|x|-x \quad h(x)=x(|x|-1)$



$D_f=[-1\ \ 0]\cup[1,+\infty)$

$f(x) \to \sqrt{x^2-x} \quad 1\le x$

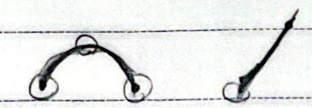
$\quad \to \sqrt{-x^2-x} \quad -1\le x\le 0$

$\longrightarrow f'(x) = \longrightarrow \dfrac{2x-1}{2\sqrt{x^2-x}} \quad 1<x \qquad x = \boxed{\tfrac{1}{2},\ 0,\ 1}$ غ ق ق

$\quad \to \dfrac{-2x-1}{2\sqrt{-x^2-x}} \quad -1< x<0 \qquad \boxed{-\tfrac{1}{2}},\ 0,\ -1$ ق ق غ

مساوی صفر نداریم چرا که مرز بازه ها مشتق را حذف می کنیم.

	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	
$f(x)$		↗	↘			↗
$f'(x)$	$+\infty$ +	0	− $-\infty$	0	$+\infty$	+



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۴ نقطه بحرانی K

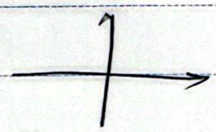
m نسبی Max = ①

n نسبی Min = 0

$f' \neq 0$ / f'_{∞} = ①

✓

⑨ $y=\dfrac{mx+2}{x-1+m}$ $\quad x=K$



$y=1-m$

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$y'=\dfrac{-m+m^2-2}{(x-1+m)^2}$ $\quad f'(x)<0 \to ad-bc<0 \to m^2-m-2<0 \to (m-2)(m+1)<0 \to -1<m<2,\ m\neq 2 \to -1<m<2$ (I)

ریشه مخرج $= x = 1-m<1 \to m>0$ (II) $\qquad$ (I)∩(II) $\to m=0,1$

⑩ $f(x)=\dfrac{x}{1-x|x|}$ $\quad \begin{cases} \dfrac{x}{1-x^2} & x\ge 0 \\ \dfrac{x}{1+x^2} & x<0 \end{cases}$ $\qquad f'(x)=\begin{cases} \dfrac{(1-x^2)-(-2x)(x)}{(1-x^2)^2} & x\ge 0 \\ \dfrac{(1+x^2)-(2x)(x)}{(1+x^2)^2} & x<0 \end{cases}$

$D_f=R-\{1\}$

$1-x^2+2x^2=0 \longrightarrow$ بدون جواب

$\dfrac{1+x^2-2x^2}{\ }=0 \qquad 1-x^2=0 \qquad x=\pm 1 \longrightarrow$ نقطه منفی جواب دارد $\longrightarrow$ فقط ① نقطه بحرانی دارد

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✓