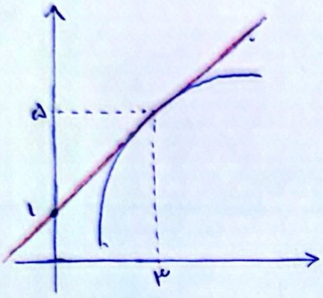
نام و نام خانوادگی ....[illegible]....  پاسخنامه تشریحی تکلیف شماره ..۲۴.. کلاس ....[illegible]....

۱




$f'(3) = \frac{5-1}{3} = \frac{4}{3}$

۲

$f(x) = \sqrt{ax-1} \rightarrow f'(x) = \frac{a}{2\sqrt{ax-1}}$

شیب : $\frac{1-2}{-1-2} = \frac{1}{3}$

معادله خط مماس : $y = \frac{x+4}{3}$

$\Rightarrow \frac{x+4}{3} = \sqrt{ax-1} \quad , \quad \frac{a}{2\sqrt{ax-1}} = \frac{1}{3}$

$\Rightarrow 9a = 2x+8 \rightarrow 5a = \frac{10x+40}{9}$

$f(5) = \sqrt{5a-1} = \sqrt{\frac{50+40}{9} - \frac{9}{9}} = \sqrt{\frac{81}{9}} = \sqrt{9} = 3$

۳

$f(x) = \frac{x^2+mx+1}{x+3} \qquad y = \frac{3x+n}{4}$

$f(1) = \frac{3+n}{4} \Rightarrow \frac{2+\overset{-2}{\cancel{m}}}{\cancel{4}} = \frac{3+n}{\cancel{4}}$

$\boxed{n=-3}$

$f'(x) = \frac{(2x+m)(x+3)-(x^2+mx+1)}{(x+3)^2} = \frac{x^2+6x+3m-1}{(x+3)^2}$

$f'(1) = \frac{3}{4} \Rightarrow \frac{\cancel{1}+6+3m-\cancel{1}}{\frac{16}{4}} = \frac{3}{4} \Rightarrow 6+3m = 12 \Rightarrow \boxed{m=-2}$

$m+n = -5$

۴

$f(x) = \frac{27-\sin^3 x}{9-\sin^2 x} = \frac{(\cancel{3-\sin x})(9+3\sin x+\sin^2 x)}{(\cancel{3-\sin x})(3+\sin x)}$

$(3g-f)(x) = \frac{\cancel{9}}{\sin x+3} - \frac{\sin^2 x+3\sin x+\cancel{9}}{3+\sin x}$

$= \frac{-\sin^2 x-3\sin x}{3+\sin x} \rightsquigarrow (3g-f)'(x) = \frac{(-2\sin x\cos x-3\cos x)(3+\sin x)-(\cos x)(-3\sin x-\sin^2 x)}{(3+\sin x)^2}$

$= \frac{-\cos x\,\cancel{(\sin^2 x+6\sin x+9)}}{\cancel{(3+\sin x)^2}} = -\cos x \qquad (3g-f)'\left(\frac{5\pi}{4}\right) = -\cos\frac{5\pi}{4} = \frac{-1}{2}$

۵

ابتدا ضابطه fog را تشکیل می‌دهیم:

$fog(x) = \frac{-1}{\sqrt[5]{\frac{1}{x^5+|x^5|}+\left|\frac{1}{x^5+|x^5|}\right|}} = \frac{-1}{\sqrt[5]{\frac{2}{x^5+|x^5|}}}$

$= \frac{-1}{\sqrt[5]{\frac{1}{x^5}}} = \frac{-1}{\frac{1}{x}} = -x \qquad (fog)'(x) = -1 \rightarrow (fog)'(\sqrt[5]{3}) = -1$

| | |
|---|---|
| $f(x)=\left(\frac{-1+\sin x}{1+\sin x}\right)^2 \quad g(x)=\frac{f(x)-1}{x}=\frac{\frac{\sin^2 x-2\sin x+1}{\sin^2 x+2\sin x+1}-1}{x} \Rightarrow$ <br> $g(x)=\frac{-4\sin x}{x(\sin^2 x+2\sin x+1)} \Rightarrow \lim_{x\to 0} g(x)=\lim_{x\to 0}\frac{-4\sin x}{x(\sin x+1)^2}$ <br> $\xrightarrow{\text{هم‌ارزی}} \frac{-4x}{x(x+1)^2}=\frac{-4}{1}=-4$ | ۶ |
| $y=x^2+1 \xrightarrow[\text{نسبت به محور x}]{\text{قرینه}} y=-x^2-1 \quad d: y=c$ <br> $-x^2-1=c \to x^2=-1-c \to x=\pm\sqrt{-1-c}$ <br> $y'=-2x \quad \left.\begin{array}{l} m_1=-2\sqrt{-1-c} \\ m_2=2\sqrt{-1-c}\end{array}\right\} m_1\times m_2=-1 \quad -4(-1-c)=-1$ <br> $-1-c=\frac{1}{4} \to c=-\frac{5}{4} \quad$ فاصله $=\frac{5}{4}$ | ۷ |
| $f(x)=2\sqrt{x}(4x^2+3) \to f'(x)=\frac{20x^2+3}{\sqrt{x}}$, $d: y=ax$ $\Rightarrow \frac{20x^2+3}{\sqrt{x}}=a \Rightarrow a\sqrt{x}=20x^2+3$ , $ax=2\sqrt{x}(4x^2+3) \Rightarrow a\sqrt{x}=8x^2+6$ <br> $\Rightarrow 8x^2+6=20x^2+3 \to \cancel{12}x^2=\cancel{3} \to x^2=\frac{1}{4}$ <br> $x=\pm\frac{1}{2} \xrightarrow{x>0} x=\frac{1}{2} \rightsquigarrow a=\frac{20(\frac{1}{2})^2+3}{\sqrt{\frac{1}{2}}} \Rightarrow a=8\sqrt{2}$ | ۸ |
| $d: y=ax \quad f(x)=\frac{\sqrt{x}}{-2x^2+x+1}=ax \to f'(x)=\frac{6x^2-x+1}{2\sqrt{x}(-2x^2+x+1)^2}=a$ <br> $a\sqrt{x}=\frac{1}{-2x^2+x+1}$ , $a\sqrt{x}=\frac{6x^2-x+1}{2(-2x^2+x+1)^2} \Rightarrow \frac{6x^2-x+1}{2(-2x^2+x+1)^{\cancel{2}}}=\frac{1}{\cancel{-2x^2+x+1}}$ <br> $6x^2-x+1=-4x^2+2x+2 \to 10x^2-3x-1=0 \quad x=\frac{1}{2}$ <br> $x=-\frac{1}{5}$ غ ق ق | ۹ |
| $f(x)=(x[x])^3 \to f'(x)=3(x[x])^2\times[x]=3[x](x[x])^2$ <br> $g(\frac{\sqrt{5}}{2})=\frac{1}{\sqrt{(\frac{\sqrt{5}}{2})^2-1}}=2 \to f(g(\frac{\sqrt{5}}{2}))=f(2)=96$ <br> $g'(\frac{\sqrt{5}}{2})=\frac{-\frac{\sqrt{5}}{2}}{((\frac{\sqrt{5}}{2})^2-1)^{\frac{3}{2}}}=1/8 \to g'(\frac{\sqrt{5}}{2})=\frac{-\frac{\sqrt{5}}{2}}{1/8}=-4\sqrt{5}$ <br> $f(g(x))'=g'(x)f'(g(x)) = 96\times -4\sqrt{5} = -384\sqrt{5}$ <br> $\frac{-384\sqrt{5}}{-48\sqrt{5}}=8$ | ۱۰ |