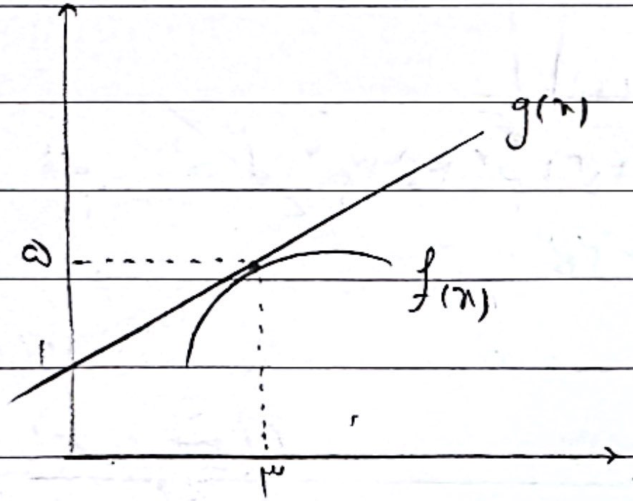
تمرین

1)



$(x - x_0)(m) = y - y_0 \rightarrow 3m = 4 \rightarrow m = \frac{4}{3}$

$g(x) = ax + b \xrightarrow{b=1} g(x) = \frac{4}{3}x + 1$

$\rightarrow$ f و g در $x=3$ برهم مماس‌اند $\rightarrow f'(3) = g'(3) = \frac{4}{3}$

سحر تیمی

۲) $(-1,1)$, $(2,2)$ $\longrightarrow m(x-x_0)=y-y_0 \rightarrow m=\frac{1}{3} \rightarrow g(x)=\frac{x}{3}+b \xrightarrow[b=\frac{4}{3}]{(-1,1)} g(x)=\frac{x+4}{3}$

مماس اند f, g : $f=g$, $\Delta=0 \rightarrow \frac{x+4}{3}=\sqrt{ax-1} \rightarrow ax-1=\frac{(x+4)^2}{9} \rightarrow$

$9ax-9=(x+4)^2 \rightarrow x^2+8x+16-9ax+9=0 \rightarrow x^2+x(8-9a)+25=0 \xrightarrow{\Delta=0}$

$(8-9a)^2+(-4)(1)(25)=0 \rightarrow (8-9a)^2=100$ ↗ $8-9a=10 \rightarrow a=\frac{-2}{9}$ ↘ $8-9a=-10 \rightarrow a=2$

$f'(x)=\frac{a}{2\sqrt{ax-1}}$ مماس $f, g \rightarrow f'=g'$ در نقطه مماس

$a=2$ ↗ $\frac{1}{\sqrt{2x-1}}=\frac{1}{3} \rightarrow ✓$

$a=-\frac{2}{9}$ ↘ $\frac{-\frac{2}{9}}{2\sqrt{\frac{-2x}{9}-1}}=\frac{-1}{9\sqrt{\frac{-2x}{9}-1}}=\frac{1}{3} \rightarrow \times$

طرف دوم مثبت
طرف اول به دلیل منفی است

$\frac{1}{\sqrt{2x-1}}=\frac{1}{3} \xrightarrow{\text{تبدیل}} 2x-1=9 \rightarrow x=5$

$f(5)=g(5)=\frac{9}{3}=3$

۳) $y=\frac{x^2+mx+1}{x+3} \xrightarrow[\text{مماس بر}]{\text{در } x=1} 4y-3x=n \rightarrow m+n=?$

$f'(x)=\frac{(2x+m)(x+3)-(x^2+mx+1)}{(x+3)^2} \xrightarrow{x=1} f'(1)=\frac{(2+m)(4)-(2+m)}{16}=\frac{6+3m}{16}$

$g(x)=\frac{3x}{4}+\frac{n}{4} \rightarrow g'(x)=\frac{3}{4}$

$f'(1)=g'(1) \rightarrow \frac{\cancel{6}^{2}+\cancel{3}m}{\cancel{16}_{4}}=\frac{\cancel{3}}{\cancel{4}} \rightarrow 4=2+m \rightarrow m=2$

s.a.m

$$f(x)=\frac{x^2+2x+1}{x+3}$$

$$f(1)=g(1)\rightarrow \frac{4}{4}=1=\frac{3+n}{4}\rightarrow n=1$$

$$g(x)=\frac{3x+n}{4}$$

$$m+n=2+1=3$$

۴) $$f(x)=\frac{27-\sin^3 x}{9-\sin^2 x}\rightarrow f(x)=\frac{\cancel{(3-\sin)}(9+\sin^2+3\sin)}{\cancel{(3-\sin)}(3+\sin)}=\sin+\frac{9}{3+\sin}$$

$$g(x)=\frac{3}{3+\sin x}$$

$$f'(x)=\cos x-\frac{9\cos}{(3+\sin)^2}\xrightarrow{\frac{5\pi}{3}}\frac{1}{2}-\frac{4.5}{(3-\frac{\sqrt3}{2})^2}$$

$$3g'\left(\frac{5\pi}{3}\right)-f'\left(\frac{5\pi}{3}\right)=?$$

$$g'(x)=\frac{-3\cos}{(3+\sin)^2}\xrightarrow{x=\frac{5\pi}{3}}\frac{-1.5}{(3-\frac{\sqrt3}{2})^2}$$

$$\hookrightarrow \cancel{\frac{-4.5}{(3-\frac{\sqrt3}{2})^2}}-\frac{1}{2}+\cancel{\frac{4.5}{(3-\frac{\sqrt3}{2})^2}}=-\frac{1}{2}$$

۵) $$f(x)=\frac{-1}{\sqrt[5]{x+|x|}}\xrightarrow{x>0}f(x)=\frac{1}{\sqrt[5]{2x}}=(2x)^{-\frac{1}{5}}$$

$$fog=\left(2\times\frac{1}{2}\times x^{-5}\right)^{-\frac{1}{5}}=x$$

$$g(x)=\frac{1}{x^5+|x^5|}\xrightarrow{x>0}g(x)=\frac{1}{2x^5}=(2x^5)^{-1}$$

$$(fog)'(x)=1$$

$$g'(\sqrt[5]{2})\,f'(g(\sqrt[5]{2}))=?\rightarrow (fog)'(x)=?\ 1$$

۶) $$f(x)=\left(\frac{\sin-1}{1+\sin}\right)^2\rightarrow\left(\frac{(\sin-1)(\sin+1)}{(1+\sin)^2}\right)^2=\left(\frac{\sin^2-1}{(1+\sin)^2}\right)^2=\left(\frac{-\cos^2 x}{(1+\sin)^2}\right)^2=$$

$$f(x)=xg(x)+1$$

$$f(x)=\left(\frac{\cos}{1+\sin}\right)^4=xg(x)+1\rightarrow g(x)=\frac{\tan^4\frac{x}{4}-1}{x}$$

$$\lim_{x\to0}g(x)=?$$

$$\frac{-1}{0}\quad \begin{matrix}0^+\to-\infty\\0^-\to+\infty\end{matrix}$$ حد ندارد

s.a.m

۷) $y = x^2+1$ $\xrightarrow[\text{نسبت به محورها}]{\text{قرینه}}$ $y=d$ خط در ۳ نقطه متقاطع ← مماس‌ها در ... ← فاصله d از (۰و۰)

۳ نقطه متقاطع‌اند

$\xrightarrow[\text{محور}x]{\text{قرینه نسبت}}$ $-y = x^2+1 \to y = -x^2-1$ $\qquad -x^2-1 = d \to -1+d = x^2 \to x = \pm\sqrt{d-1}$

$y' = 2x \quad \nearrow y' = 2\sqrt{d-1} \qquad 2\sqrt{d-1} = \dfrac{1}{2\sqrt{d-1}} \to 4(d-1) = 1 \to d-1 = \dfrac{1}{4}$

$\qquad\quad \searrow y' = -2\sqrt{d-1}$

$d = \dfrac{5}{4}$

$y = \dfrac{5}{4}$ $\qquad$ فاصله از $(0,0)$ $= \dfrac{5}{4}$

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۸) $g(x) = kx \to$ خط d $\to g'(x) = k$

$f(x) = 2\sqrt{x}\,(4x^2+3) \to f(x) = 8x^{\frac{5}{2}} + 6x^{\frac{1}{2}} \to f'(x) = 20x^{\frac{3}{2}} + 3x^{-\frac{1}{2}}$

$f(x) = g(x)\ ,\ \Delta = 0 \to kx = 2\sqrt{x}\,(4x^2+3) \to k = \dfrac{8x^2+6}{\sqrt{x}}$

$g'(x) = f'(x) \to k = 20x^{\frac{3}{2}} + 3x^{-\frac{1}{2}}$

$20x\sqrt{x} + \dfrac{3}{\sqrt{x}} = \dfrac{8x^2+6}{\sqrt{x}} \xrightarrow{\times\sqrt{x}} 20x^2+3 = 8x^2+6$

$12x^2 = 3 \to x = \pm\dfrac{1}{2}$

$D_f: x \geq 0 \to \begin{cases} x = \frac{1}{2} \ \checkmark \\ x = -\frac{1}{2} \ \times \end{cases}$

$g(\frac{1}{2}) = f(\frac{1}{2}) \to \dfrac{9}{\sqrt{2}} = \dfrac{k}{2} \to \dfrac{18\sqrt{2}}{2} = 9\sqrt{2} = k$

$\to y = 9\sqrt{2}\,x \to$ شیب خط d $= 9\sqrt{2}$

s.a.m

9) $f(x) = \dfrac{\sqrt{x}}{-2x^2 + x + 1}$

$d: g(x) = kx$ خط

$\rightarrow$ مماس در A $\rightarrow$ x کدام ؟

$A(a, f(a))$ $\xrightarrow[\text{شیب}]{\text{مبدا}}$ $m_d = \dfrac{f(a) - 0}{a - 0} = \dfrac{f(a)}{a}$

$f'(a) = g'(a) = \dfrac{f(a)}{a} \rightarrow \dfrac{\left(\frac{1}{2\sqrt{x}}\right)(-2x^2 + x + 1) - (-4x + 1)(\sqrt{x})}{(-2x^2 + x + 1)\cancel{(-2x^2+x+1)}} = \dfrac{\sqrt{x}}{\sqrt{x}\,(x)\cancel{(-2x^2+x+1)}}$

$10x^2 - 3x - 1 = 0 \rightarrow x^2 - 3x - 10 = 0 \rightarrow (x-5)(x+2) = 0 \quad \begin{cases} x = \frac{5}{10} = \frac{1}{2} & \checkmark \\ x = \frac{-2}{10} = \frac{-1}{5} & \times \end{cases}$

$f\left(\frac{1}{2}\right) = \dfrac{\frac{1}{\sqrt{2}}}{\frac{-1}{2} + \frac{1}{2} + 1} = \dfrac{\sqrt{2}}{2}$

با توجه به دامنه $x \geq 0$

10) $f(x) = (x[x])^3 \rightarrow f'(x) = 3([x])(x[x])^2$

$g(x) = \dfrac{1}{\sqrt{x^2 - 1}} = (x^2-1)^{-\frac{1}{2}} \qquad g'(x) = -\frac{1}{2}(x^2-1)^{-\frac{3}{2}} \times 2x \xrightarrow{x = \frac{\sqrt{5}}{2}} g'(x) = -4\sqrt{5}$

$x < \frac{\sqrt{5}}{2} \rightarrow x^2 < \frac{5}{4} \rightarrow x^2 - 1 < \frac{1}{4} \rightarrow -\frac{1}{2} < \sqrt{x^2-1} < \frac{1}{2}$

$\dfrac{(fog)' - \left(\frac{\sqrt{5}}{2}\right)}{-48\sqrt{5}} = \rightarrow \begin{cases} g(x) > 2 \xrightarrow{g(x) > 0} \checkmark \rightarrow g\left(\frac{\sqrt{5}}{2}\right) = 2^+ \\ g(x) < -2 \xrightarrow{g(x) > 0} \times \end{cases}$

$f'(g(x)) = f'(2^+) = 3(2)(4)^2 = 96$

$\dfrac{(fog)'\left(\frac{\sqrt{5}}{2}\right)^-}{-48\sqrt{5}} = \dfrac{f'\left(g\left(\frac{\sqrt{5}}{2}\right)^-\right) \times g'\left(\frac{\sqrt{5}}{2}\right)}{-48\sqrt{5}} = \dfrac{96 \times -4\sqrt{5}}{-48\sqrt{5}} = 8$