۱۴٫۲۵

پردیس ایرانی ۱

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$g'(x)=f'(\tan^2 x+\sqrt{2}\cos x)(2\tan x(1+\tan^2 x)-\sqrt{2}\sin x)$

$x=\frac{\pi}{4}\Rightarrow g'(\frac{\pi}{4})=f'(2)(3)=\sqrt{3}\Rightarrow f'(2)=\frac{\sqrt{3}}{3}$

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$(f-2g)'(\frac{7\pi}{6}) \Rightarrow \frac{8+\cos^3 x}{4-\cos^2 x}-\frac{4}{2-\cos x}=\frac{8+\cos^3 x-4(2+\cos x)}{4-\cos^2 x}$

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$=\frac{\cos^3 x-4\cos x}{4-\cos^2 x}=\frac{\cos x(\cos^2 x-4)}{4-\cos^2 x}=-\cos x \xrightarrow{\text{مشتق}} \sin x \Rightarrow \sin\frac{7\pi}{6}=-\frac{1}{2}$

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$x>\frac{4}{3}: \quad \frac{(4x-3)a}{2\sqrt{ax}}+4\sqrt{ax}$

$x<\frac{4}{3}: \quad \frac{-a(4x-3)}{2\sqrt{ax}}-4\sqrt{ax}$

$\Rightarrow \frac{(4x-3)a}{\sqrt{ax}}+8\sqrt{ax} \Rightarrow 8\sqrt{ax}=2\sqrt{6}$

$4\sqrt{ax}=\sqrt{6}$

$\cancel{3}\times\frac{4}{\cancel{3}}\times a=6$

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$a=\frac{6}{12}=\frac{1}{2}$

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$y=ax+2 \xrightarrow{x=4} y=-4a+2 \qquad -4a+2-a=-1 \Rightarrow -5a=-3$

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$f'(4)=-a=-\frac{3}{5}=-0.6 \qquad a=\frac{3}{5}$

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$7y-x=5 \Rightarrow y=\frac{x+5}{7} \qquad \frac{x+5}{7}=\frac{ax-1}{3x+1}\Rightarrow 7(ax-1)=(x+5)(3x+1)$

$\Rightarrow 7ax-7=3x^2+x+15x+5 \Rightarrow 3x^2+(16-7a)x+12=0$

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$\Delta=0 \Rightarrow (16-7a)^2-4(12)(3) \Rightarrow 16-7a=\pm 12 \Rightarrow a=\frac{4}{7} \mid \boxed{4}$

$\downarrow \qquad \downarrow$

غ ق ق $\quad x=-2 \qquad x=2$

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$x=2\Rightarrow 8+2a-b=0\Rightarrow 2a-b=-8$

$f'(x)=3x^2+a \xrightarrow{x=2} 12+a=0\Rightarrow a=-12$

$\Rightarrow -24-b=-8\Rightarrow b=-16$

$b-a=-16-(-12)=-4$

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$f(x)=\sin^n(x^2)$ $\quad f'(0)=\lim_{x\to 0}\frac{f(x)-f(0)}{x}=\frac{\sin^n(x^2)}{x}=\frac{x^{2n}}{x}=x^{2n-1}$ ۷-

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$\lim_{x\to 0}\frac{f(x)f'(x)}{(1-\cos x)^m}=\frac{\sin^n(x^2)\,x^{2n-1}}{(1-\cos x)^m}=\frac{x^{4n-1}}{x^{2m}\times 2^{-m}}\Rightarrow x^{4n-2m-1}\times 2^{m+1}=32\sqrt{2}=2^5\times 2^{\frac{1}{2}}=2^{\frac{11}{2}}$

$m=\frac{7}{2}$ $\quad 2m+n=9$

$4n=2m+1\Rightarrow 4n=12 \quad n=3 \quad 2m+n=11+3=14$ $\quad n=2$

$2^m=2^{\frac{11}{2}}\Rightarrow m=\frac{11}{2}$ $\quad \begin{cases}4n-2m-1=0\longrightarrow n=\frac{2m+1}{4}\\ 2^{m+1}n=32\sqrt{2}\end{cases}\longrightarrow 2^{m+1}\times\frac{2m+1}{4}=32\sqrt{2}\longrightarrow(2m+1)2^{m+1}=2^7\sqrt{2}$

$f(x)=\frac{\sqrt{x}+1}{\sqrt{x}-1}\Rightarrow\frac{\sqrt{y}+1}{\sqrt{y}-1}=x\Rightarrow\sqrt{y}=\frac{x+1}{x-1}\Rightarrow y=\left(\frac{x+1}{x-1}\right)^2$ ۸-

$\Rightarrow y'=\frac{-4x^2+4}{(x-1)^4}\xrightarrow{x=2}y'=\frac{-4(4)+4}{1}=-12$

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$f(x)=(x^2+1)^3(ax+1)\rightarrow f(0)=1 \quad f(-1)=8(-a+1)$ ۹-

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$\frac{1-8(-a+1)}{0-(-1)}=\frac{1+8a-8}{1}=-11\Rightarrow -8a=4 \quad a=-\frac{1}{2} \quad x=-2a=1$

$f'(x)=3(x^2+1)^2(2x)(-\frac{1}{2}x+1)+(-\frac{1}{2})(x^2+1)^3\xrightarrow{x=1}12-4=8$

$y=\sin x\cos 2x\Rightarrow x=\frac{\pi}{4}\rightarrow y=\frac{\sqrt{2}}{2}\times 0=0 \quad x=\frac{\pi}{2}\Rightarrow y=1\times(-1)=-1$ ۱۰-

$\frac{-1-0}{\frac{\pi}{2}-\frac{\pi}{4}}=\frac{-1}{\frac{\pi}{4}}=-\frac{4}{\pi}$

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$y=\sin^4x-\cos^4x\rightarrow x=\frac{\pi}{4}\rightarrow y=0 \quad x=\frac{\pi}{2}\rightarrow y=1-0=1$

$\frac{1-0}{\frac{\pi}{2}-\frac{\pi}{4}}=\frac{1}{\frac{\pi}{4}}=\frac{4}{\pi} \qquad \frac{-\frac{4}{\pi}}{\frac{4}{\pi}}=-1$