(فاطمه شعبانی)

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تست ۲۳

1)

$\hookrightarrow$ if $x = \frac{\pi}{4} \to \underbrace{\tan^2(\frac{\pi}{4})}_{1} + \underbrace{\sqrt{2}\cos(\frac{\pi}{4})}_{\sqrt{2}\times\frac{\sqrt{2}}{2} = 1} = 2$

$\hookrightarrow g(\frac{\pi}{4}) = f(2) \to g'(\frac{\pi}{4}) = f' \to$

$g'(\frac{\pi}{4}) = \underbrace{(2\times\tan x\times(1+\tan^2 x)) + (-\sqrt{2}\sin x)}\times f'(2) \Rightarrow$

$x = \frac{\pi}{4} \to \underbrace{2\times1\times2}_{4} + \underbrace{-\sqrt{2}\times\frac{\sqrt{2}}{2}}_{-1} = 3$

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$\hookrightarrow \sqrt{3} = 3\times f'(2) \to \boxed{f'(2) = \frac{\sqrt{3}}{3}}$

۴) $f(x) = \dfrac{۸+\cos^۳ x}{۴-\cos^۲ x}$, $g(x) = \dfrac{۲}{۲-\cos x}$ $\rightarrow f'(\frac{۷\pi}{۶}) - ۲g'(\frac{۷\pi}{۶}) = ?$

$(f-۲g)'(\frac{۷\pi}{۶}) \Rightarrow$

$* \;\dfrac{(\cancel{۲+\cos x})(۴-۲\cos x+\cos^۲ x)}{(۲-\cos x)(\cancel{۲+\cos x})} = \dfrac{۴-۲\cos x+\cos^۲ x}{۲-\cos x}$ ۴

$\rightarrow \dfrac{\cancel{۴}-۴\cos x+\cos^۲ x-\cancel{۴}}{۲-\cos x} = \dfrac{\cos x(\cos x - \cancel{۲})}{\cancel{۲-\cos x}} = -\cos x$

$\rightarrow (-\cos x)' = \sin(x) = \sin(\frac{۷\pi}{۶}) = \boxed{\frac{-۱}{۲}}$ ✓

$*$ ۳) $|۴x-۳|\sqrt{ax}$ $\quad x=\frac{۳}{۴} \rightarrow ۲\sqrt{۶}$

$\frac{۳}{۴}$: $-\;|\;+$

مشتق از راست $\rightarrow x=\frac{۳}{۴} \rightarrow (۴x-۳)\sqrt{ax}$ (I)

مشتق از چپ $\rightarrow x=\frac{۳}{۴} \rightarrow (۳-۴x)\sqrt{ax}$ (II)

$(I)' = ۴\sqrt{ax} + \dfrac{a}{۲\sqrt{ax}}\underbrace{(۴x-۳)}_{۰} \rightarrow ۴\sqrt{\frac{۳}{۴}a}$ ۲

$(II)' = -۴\sqrt{ax} + \dfrac{a}{۲\sqrt{ax}}\underbrace{(۳-۴x)}_{۰} \rightarrow -۴\sqrt{\frac{۳}{۴}a}$

$\sqrt{\frac{۳}{۴}a}\times ۴ - (-۴\sqrt{\frac{۳}{۴}a}) = \cancel{۸}\sqrt{\frac{۳}{۴}a} = \cancel{۲}\sqrt{۶} \rightarrow \sqrt{\frac{۳}{۴}a} = \sqrt{\frac{۶}{۱۶}} = \sqrt{\frac{۳}{۸}}$

$\rightarrow \frac{۳}{۴}a = \frac{۳}{۸} \rightarrow \boxed{a=\frac{۱}{۲}}$ ✓

۴) $y+ax=2 \to x=4 \to$ f بر مماس

$f(4)+f'(4)=1 \to f'(4)=?$

$y=-ax+2 \to f'(4)=-a$ ⎫ $\to -4a+2-a=-1$

$\hookrightarrow y=f(4)=-4a+2$ ⎭ $-5a=-3 \to a=\frac{3}{5}$

$\hookrightarrow f'(4)=-a=\boxed{-\frac{3}{5}}$

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۵) $7y-x=5 \to \frac{ax-1}{3x+1} \to a=?$

معادله تلاقی → $\Delta=0$

$\hookrightarrow \frac{ax-1}{3x+1}=\frac{5+x}{7} \to 7ax-7=3x^2+16x+5$

$\to 3x^2+(-7a+16)x+12=0 \xrightarrow{\Delta} (16-7a)^2-144=0$

$\to |16-7a|=12 \to a=\frac{4}{7}$, $a=4$ $\to$ باید بررسی بشه!

$a=4 \to 3x^2-12x+12=0 \to 3(x-2)^2=0 \to x=2$ ✓ قق

$\hookrightarrow \boxed{a=4}$

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۶) x ریشه تکراری $\to 3x^2+a=0 \to 12+a=0 \to \boxed{a=-12}$

$\hookrightarrow x^3+ax-b=0 \to \underbrace{8-24}_{-16}-b=0$ $\boxed{b=-16}$

$\to b-a=-16+12=\boxed{-4}$

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۷) $f(x)=\sin^n(x^2)$ $\quad \lim \dfrac{f(x)\,f'(x)}{(1-\cos x)^m}=۳۲\sqrt{۲}$

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$۲m+n \to ?$

$\to$ حد $\frac{0}{0}$ $\to$ هوپیتال $\to$

$$\lim_{x\to 0}\frac{\sin^n(x^2)\times n\sin^{n-1}(x^2)\times\cos(x^2)\times(2x)}{(2\sin^2\frac{x}{2})^m}\xrightarrow{\text{هم‌ارزی}}\lim_{x\to 0}\frac{(x^2)^n\,n\,(x^2)^{n-1}\times 2x}{\left(2\left(\frac{x}{2}\right)^2\right)^m}=$$

$$\lim_{x\to 0}\frac{2nx^{2n+2n-2+1}}{\left(\frac{x^2}{2}\right)^m}=\lim_{x\to 0}\frac{2nx^{4n-1}}{\frac{1}{2^m}x^{2m}}=\lim_{x\to 0}2^{m+1}\,n\,x^{4n-2m-1}$$ $\rightsquigarrow$ حاصل حد عدد غیرصفر متناهی غیرصفر ← توان صفر شوند

$$\begin{cases}4n-2m-1=0\to n=\frac{2m+1}{4}\\ 2^{m+1}n=۳۲\sqrt{۲}\end{cases}\longrightarrow 2^{m+1}\times\frac{2m+1}{4}=۳۲\sqrt{۲}\to(2m+1)2^{m+1}=2^7\sqrt{۲}\to m=\frac{7}{2},\quad n=2$$

$2m+n=۹$

۸) $\to y=\dfrac{\sqrt{x}+1}{\sqrt{x}-1}\to y\sqrt{x}-y=\sqrt{x}+1$

$y\sqrt{x}-\sqrt{x}=y+1\to\sqrt{x}(y-1)=y+1$

$\to\sqrt{x}=\dfrac{y+1}{y-1}\to x=\left(\dfrac{y+1}{y-1}\right)^2\xrightarrow{f^{-1}}y=\left(\dfrac{x+1}{x-1}\right)^2$

$\to y=\dfrac{x^2+2x+1}{x^2-2x+1}\xrightarrow{x=2}y'\Rightarrow\dfrac{(2x+2)(x^2-2x+1)-(2x-2)(x^2+2x+1)}{(x^2-2x+1)^2}$

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$=\dfrac{(6\times1)-(2\times9)}{1}=6-18=\boxed{-12}$

۹) $f(x) = (x^2+1)^3(ax+1) \to [-1,0] \to -11$

$\hookrightarrow \frac{f(0)-f(-1)}{0-(-1)} = -11 \to \frac{1-(8\times\overbrace{(-a+1)}^{-8a+8})}{1} = -11$

$\to 1+\underbrace{8a-8}_{8a-7} = -11 \to 8a=-4 \to a=-\frac{1}{2}$

$\to x = -2a = -2\times -\frac{1}{2} = 1$

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$\to f'(1) \Rightarrow (x^2+1)^3(-\frac{1}{2}x+1) \to$

$3(x^2+1)^2(2x)(-\frac{1}{2}x+1) + (-\frac{1}{2})(x^2+1)^3 =$

$\underbrace{3\times 4\times \cancel{2}\times\frac{1}{\cancel{2}}}_{12} + (-\frac{1}{\cancel{2}}\times\cancel{8}) = \boxed{8}$

$12 + -4$

۱۰) $\to \frac{f(\frac{\pi}{2})-f(\frac{\pi}{4})}{\frac{\pi}{2}-\frac{\pi}{4}} = \frac{-1-(0)}{\frac{\pi}{4}} = \frac{-4}{\pi}$

$y=\sin x\cos 2x$

$f(\frac{\pi}{2}) = \underbrace{\sin x}_{1}\underbrace{\cos 2x}_{-1} = -1 \qquad f(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}\times 0 = 0$

$\circ \to$

$y=\sin^2 x-\cos^2 x \to f(\frac{\pi}{2}) = 1-0 = 1$

$f(\frac{\pi}{4}) = 0$ $(\sin x=\cos x)$ $\Big\} \to \frac{1-0}{\frac{\pi}{4}} = \frac{4}{\pi}$

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$\to \frac{-\frac{4}{\pi}}{\frac{4}{\pi}} = \boxed{-1}$