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۱۴۰۳/۱۱/۲۳ تکلیف شماره ۲۳ ۱۸/۲۵ هانیه نوحیان

دوازدهم دختر A

۴) سوال چهار

خط $y + ax = 2 \xrightarrow{x=4}$ مماس ، $f(4) + f'(4) = -1$

$y = -ax + 2$

$$\left.\begin{aligned} f(4) &= -4a + 2 \\ f'(4) &= -a \end{aligned}\right\} \Rightarrow \begin{aligned} -4a + 2 - a &= -1 \\ -5a &= -3 \end{aligned}$$

$$a = \frac{3}{5} \Rightarrow \boxed{f'(4) = -\frac{3}{5}}$$

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۵) سوال پنج

$y = \frac{x+5}{7}$ منحنی $\Rightarrow \frac{ax-1}{3x-1}$

برای نقطه‌ی تماس این دو باید فقط ۱ جواب مشترک داشته باشد $\Leftarrow$

$$\frac{x+5}{7} = \frac{ax-1}{3x+1} \Rightarrow (x+5)(3x+1) = 7(ax-1) \Rightarrow$$

$3x^2 + (16 - 7a)x + 12 = 0 \xRightarrow[\text{برای مماس بودن}]{\Delta = 0} (16-7a)^2 - 4(3)(12) = 0 \Rightarrow (16-7a)^2 - 144 = 0$

$\Rightarrow (16-7a)^2 = 144 \Rightarrow$ ۲ حالت داره

① $16 - 7a = 12 \rightarrow a = \frac{4}{7}$ چون نقطه در صورت سوال

② $16 - 7a = -12 \rightarrow a = 4$ در ناحیه اول مختصات

$x_0 = \frac{7a - 16}{6} \leftarrow x > 0$

$x_0 = 2 \leftarrow a = 4$

$\boxed{a = 4}$

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$$g(x) = f(\tan^2 x + \sqrt{2}\cos x)$$ ۱) سوال یک

$g'(\frac{\pi}{4}) = \sqrt{2}$ $f'(2) = ?$ $u(x) = \tan^2 x + \sqrt{2}\cos x \Rightarrow$

$g(x) = f(u(x)) \Rightarrow g'(x) = f'(u(x)) \cdot u'(x) \Rightarrow u(\frac{\pi}{4}) \Rightarrow \tan(\frac{\pi}{4}) = 1$

$\cos(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}$

$u'(\frac{\pi}{4}) = (1)^2 + \sqrt{2} \cdot \frac{\sqrt{2}}{2} = 1 + 1 = 2 \Rightarrow g'(\frac{\pi}{4}) = f(2) \cdot u'(\frac{\pi}{4})$

$\Rightarrow u'(x) = \tan^2 x + \sqrt{2}\cos x \Rightarrow \frac{d}{dx}(\tan^2 x) = 2\tan x \sec^2 x$

$\frac{d}{dx}(\sqrt{2}\cos x) = -\sqrt{2}\sin x \Rightarrow u(x) = 2\tan x \sec^2 x - \sqrt{2}\sin x$ ۲

$\Rightarrow x = \frac{\pi}{4} \Rightarrow \tan\frac{\pi}{4} = 1$ $\sec\frac{\pi}{4} = \frac{1}{\cos\frac{\pi}{4}} = \sqrt{2} \Rightarrow \sec^2\frac{\pi}{4} = 2$

$\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$ $u(\frac{\pi}{4}) = 2(1)(2) - \sqrt{2} \cdot \frac{\sqrt{2}}{2} = 4 - 1 = 3$

$g'(\frac{\pi}{4}) = f'(2) \cdot 3 \Rightarrow \sqrt{3} = 3f(2) \longrightarrow \boxed{f'(2) = \frac{\sqrt{3}}{3}}$

$f(x) = \frac{8 + \cos^3 x}{4 - \cos^2 x} \xrightarrow{\cos x = t} f(t) = \frac{8 + t^3}{4 - t^2}$ ۲) سوال ۲

$g(x) = \frac{2}{2 - \cos x} \Rightarrow g(t) = \frac{2}{2 - t}$ $\frac{d}{dx} f(t) = f(t) \cdot t(t)$

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$t = -\sin x \Rightarrow f(x) = -\sin x \, f(t)$

$g(x) = -\sin x \, g(t)$

$f(x) - 2g(x) = -\sin x \left(f(t) - 2g(t) \right) \xRightarrow{t \text{ تعویض}} f(t) = \dfrac{8+t^3}{4-t^2} \Rightarrow f'(t) =$

$\dfrac{3t^2(4-t^2) - (8+t^3)(-2t)}{(4-t^2)^2} = 12t^2 - 3t^4 + 16t + 2t^4 = 12t^2 + 16t - t^4$

$f'(t) = \dfrac{12t^2 + 16t - t^4}{(4-t^2)^2}$ $\qquad g(t) = \dfrac{2}{2-t} = 2(2-t)^{-1}$

$g'(t) = \dfrac{2}{(2-t)^2}$

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$x = \frac{7\pi}{6}$

$\cos x = -\frac{\sqrt{3}}{2} \Rightarrow t = -\frac{\sqrt{3}}{2}$ $\qquad \sin x = -\frac{1}{2}$

$f - 2g = -\sin x \left(f(t) - 2g(t) \right) = \frac{1}{2} \underbrace{\left(f(t) - 2g(t) \right)}_{-1} = -\frac{1}{2}$

سوال ۳)

$f(x) = |4x-3| \sqrt{ax}$ $\qquad x_0 = \frac{3}{4}$

$4x - 3 = 0 \longrightarrow x = \frac{3}{4}$

سمت راست $\Leftarrow x > \frac{3}{4}$

$|4x-3| = 4x-3 \Rightarrow f(x) = (4x-3)\sqrt{ax}$

$\Rightarrow 4x - 3 = 0 \longrightarrow f(x_0) = 0 \longrightarrow f'_+(x_0) = (4)\sqrt{a x_0}$

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سمت چپ $x<\frac{3}{4}$

$|4x-3| = -(4x-3) = 3-4x$

$f(x) = (3-4x)\sqrt{ax} \Rightarrow f'_-(x_0) = (-4)\sqrt{ax_0}$

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اختلاف شیب ها

$|f'_+ - f'_-| = |4\sqrt{ax_0} - (-4\sqrt{ax_0})| = 8\sqrt{ax_0}$

جایگذاری $x_0 = \frac{3}{4}$

$8\sqrt{ax_0} = 2\sqrt{6}$

$8\sqrt{a\cdot\frac{3}{4}} = 2\sqrt{6} \Rightarrow 4\sqrt{3a} = 2\sqrt{6} \longrightarrow 2\sqrt{3a} = \sqrt{6} \Rightarrow 4(3a) = 6$

$12a = 6$

$a = \frac{1}{2}$

سوال ۶) $f(x) = x^3+ax-b \qquad x=2 \qquad b-a=?$

وقتی $x=2$ مماس باشه $\Leftarrow f(2)=0$

$f(2) = 2^3+2a-b=0 \Rightarrow 8+2a-b=0 \Rightarrow b = 8+2a$

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$f'(x) = 3x^2+a \Rightarrow f'(2) = 3(4)+a = 0 \Rightarrow 12+a=0 \Rightarrow a=-12$

$b = 8+2(-12) = -16 \qquad b-a = -16-(-12) = \boxed{-4}$

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سوال ۱۷

$x \to 0$ تغییر نمیکنیم

$\sin t \sim t \qquad 1-\cos x \sim \frac{x^2}{2}$

$f(x) = \sin^n(x^2) \Rightarrow x\to 0 \Rightarrow \sin(x^2) \sim x^2$

$f(x) \sim (x^2)^n = x^{2n}$

$f'(x) = n\sin^{n-1}(x^2)\cos(x^2) \cdot 2x$

$\cos(x^2) \sim 1 \qquad \sin(x^2) \sim x^2 \qquad f'(x) \sim n(x^2)^{n-1} \cdot 2x = 2n\, x^{2n-1}$

صورت $f(x) f'(x) \sim x^{2n} \cdot 2n x^{2n-1} = 2n x^{4n-1}$

$(1-\cos x)^m \sim \left(\frac{x^2}{2}\right)^m = \frac{x^{2m}}{2^m}$

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$$\frac{f(x) f'(x)}{(1-\cos x)^m} \sim \frac{2n\, x^{4n-1}}{x^{2m}/2^m} = 2n\, 2^m x^{4n-1-2m}$$

$4n-1-2m=0$

$2m = 4n-1 \Rightarrow m = 2n - \frac{1}{2}$

$2n\, 2^m = 32\sqrt{2}$

$2n \cdot 2^{2n-\frac{1}{2}} = 32\sqrt{2}$

$2n \cdot \frac{2^{2n}}{\sqrt{2}} = 32\sqrt{2}$

$2n\, 2^{2n} = 64$

$n\, 2^{2n} = 32$

$n=2 \Rightarrow m = 2n - \frac{1}{2} = 4 - \frac{1}{2} = \frac{7}{2}$

$2m+n = \frac{7}{\cancel{2}} + 2 = \cancel{\frac{11}{2}}$ 9

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سوال ۱۸) $f(x)=\frac{\sqrt{x}+1}{\sqrt{x}-1}$ $\quad (f^{-1})'(a)=\frac{1}{f'(x_0)}$

$f^{-1}(x)=\left(\frac{x+1}{x-1}\right)^2 \to (f^{-1})'=2\left(\frac{x+1}{x-1}\right)\times\frac{-2}{(x-1)^2} \longrightarrow (f^{-1}(2))'=2\times3\times(-2)=-12$

$f(x_0)=a$

$\frac{\sqrt{x}+1}{\sqrt{x}-1}=2 \quad t=\sqrt{x} \quad \frac{t+1}{t-1}=2 \quad t+1=2t-2$

$\boxed{t=3}$

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$x_0=t^2=9 \quad t=\sqrt{x} \quad f=\frac{t+1}{t-1}$

$\frac{df}{dt}=\frac{(t-1)-(t+1)}{(t-1)^2}=\frac{-2}{(t-1)^2} \quad \frac{dt}{dx}=\frac{1}{2\sqrt{x}}=\frac{1}{2t}$

$f'(x)=\frac{df}{dt}\cdot\frac{dt}{dx}=\frac{-2}{(t-1)^2}\cdot\frac{1}{2t}=\frac{1}{t(t-1)^2}$

$\Rightarrow f'(9)=-\frac{1}{3(2)^2}=\boxed{-\frac{1}{12}} \quad (f^{-1})'(2)=\frac{1}{f'(9)}=\frac{1}{12}$

$f'(x)$ غلط $\longrightarrow (f^{-1})'=-12$

سوال ۴

آهنگ تغییر متوسط $[-1,0]$ $=\frac{f(0)-f(-1)}{0-(-1)}=\frac{1-(8(-a+1))}{1}=\frac{1-(-8a+8)}{1}=8a-7=-11 \longrightarrow \begin{cases}a=-\frac{1}{2}\\ x=1\end{cases}$

$f'(x)=3(2x)(x^2+1)^2(ax+1)+a(x^2+1)^3 \xrightarrow[x=-\frac{1}{2}]{x=1} 3(2\times1)(1+1)^2\left(-\frac{1}{2}\times1+1\right)+-\frac{1}{2}(1+1)^3=8$

سوال ۱۰

$\begin{cases}y_1(\frac{\pi}{4})=\sin(\frac{\pi}{4})\cos(\frac{\pi}{2})=0\\ y_1(\frac{\pi}{2})=\sin(\frac{\pi}{2})\cos(\pi)=-1\end{cases} \longrightarrow \frac{\Delta y_1}{\Delta x}=\frac{-1-0}{\frac{\pi}{2}-\frac{\pi}{4}}=\frac{-4}{\pi}$

$\longrightarrow \frac{\frac{\Delta y_1}{\Delta x}}{\frac{\Delta y_2}{\Delta x}}=\frac{-\frac{4}{\pi}}{\frac{4}{\pi}}=-1$

$y_2=\sin^4 x-\cos^4 x=-\cos 2x$

$\begin{cases}y_2(\frac{\pi}{4})=-\cos(\frac{\pi}{2})=0\\ y_2(\frac{\pi}{2})=-\cos(\pi)=1\end{cases} \longrightarrow \frac{\Delta y_2}{\Delta x}=\frac{1-0}{\frac{\pi}{2}-\frac{\pi}{4}}=\frac{4}{\pi}$

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