کیمیایی آبادی ۱۸

①

$g(x) = f(\underbrace{\tan^2 x + \sqrt{2}\cos x}_{h(x)})$

$\rightarrow g'(x) = (2x(1+\tan^2 x)\times\tan x) \times f'(\tan^2 x + \sqrt{2}\cos x)$

$g'(\frac{\pi}{4}) = 2f'(2) = \sqrt{3} \rightarrow f'(2) = \frac{1}{\sqrt{3}} \rightarrow f'(2) = \frac{\sqrt{3}}{3}$

$x = \frac{\pi}{4} \rightarrow (2(1+1)\times 1) f'(1 + \frac{\sqrt{2}\times\sqrt{2}}{1^2}) = \sqrt{3} \rightarrow 4f'(2) = \sqrt{3} \rightarrow f'(2) = \frac{\sqrt{3}}{4}$

②

$f(x) = \frac{8 + \cos^3 x}{4 - \cos^2 x}$

$g(x) = -2 \times \frac{2(2+\cos x)}{(2-\cos x)(2+\cos^2 x)}$

$f(x) - 2g(x) = \frac{\cos^3 x - 4\cos x}{4 - \cos^2 x} - \cos x(4 - \cos^2 x)$

$= -\cos x \xrightarrow{\text{مشتق}} = \sin x$

$x = \frac{5\pi}{6} \rightarrow \sin\frac{5\pi}{6} = \frac{1}{2}$ ✓

③

$f(x) = |4x-3|\sqrt{ax} \rightarrow f'_+(x) = 4\sqrt{ax} + (4x-3)\frac{a}{2\sqrt{ax}} \xrightarrow{x=\frac{3}{4}} = \frac{4\sqrt{3a}}{2}$

$(f(x) = (4x-3)\sqrt{ax})$

$\rightarrow f'_-(x) = -4\sqrt{ax} + (-4x+3)\frac{a}{2\sqrt{ax}} \xrightarrow{x=\frac{3}{4}} = -2\sqrt{3a}$

$(f(x) = (-4x+3)\sqrt{ax})$

$2\sqrt{3a} - (-2\sqrt{3a}) = \frac{4\sqrt{3a}}{2\times 2} = 2\sqrt{7} \rightarrow a = \frac{1}{4}$ ✓

④

$y = -ax + 2$ مماس

$x = 4$ در

$f(4) + f'(4) = -1$

$-4a + 2 \quad -a$

$-3a = -3$

$\rightarrow a = \frac{3}{3}$

$f'(4) = -a = -\frac{3}{3}$ ✓

⑤

$y = \frac{1}{5}x + \frac{3}{5}$ مماس

شیب $\rightarrow \frac{1}{5}$

$y = \frac{ax-1}{3x+1} \xrightarrow{\text{مشتق}} \frac{a+3}{3x+1}$

$\frac{ax-1}{3x+1} = \frac{3+x}{5} \rightarrow 5ax - 5 = 3x^2 + 14x + 3 \rightarrow 3x^2 + (14-5a) + 12 = 0$

چون مماس است $\Delta = 0 \rightarrow (14-5a)^2 - 4\times 3\times 12 = 0$

$49a^2 - 32\times 5a + 256 - 144 = 0 \qquad a = \frac{4}{5}$ یا $a = 4$

$(5a-4)(5a-28)$

$3x^2 + (14-7a)x + 12 \xrightarrow{\text{مماس}} x = \frac{-b}{2a} = \frac{-14+7a}{6} \rightarrow \begin{cases} a = 4 \rightarrow x = 2 & \checkmark \\ a = \frac{4}{5} \rightarrow x = -2 & \times \end{cases}$

(۶)

$f(x) = x^3 + ax - b$
محور xها $\to y = 0$
$\to x^3 + ax - b = 0 \xrightarrow{x=2} 8 + 2a - b = 0$
$2a - b = -8$ ★

$f'(x) = 3x^2 + a \xrightarrow{x=2} 12 + a = 0 \to a = -12 \xrightarrow{★} -24 - b = -8$

مشتق در $x=2$ ← صفر است ← چون بر محور xها مماس است

$b = -16$

(۴)

$b - a = -16 + 12 = -4$ ✓

(۷)

$\lim_{x \to 0} \frac{f(x) f'(x)}{(1-\cos x)^m} = 32\sqrt{2}$ $f'(0) = \lim_{x \to 0} \frac{f(x) - f(0)}{x - 0} = \frac{\sin^n(x^2)}{x}$ x^{2n} $= x^{2n-1}$

$32\sqrt{2} = \frac{\sin^n(x^2)(x^{2n-1})}{(1-\cos x)^m}$ (x^{2n}, $\frac{x^2}{2}$) $= \frac{2n x^{4n-1} \times 2^m}{\frac{x^{2m}}{2^m}} = 32\sqrt{2}$

(۱)

$\xrightarrow{x=0} 2^m = 32\sqrt{2}$ $2^{\frac{11}{2}} \to m = \frac{11}{2}$ $4n - 1 = 2(m) \to n = 3$ ($\frac{11}{2}$)

$2m + n = 11 + 3 = 14$

$\lim_{x \to 0} \frac{2nx^{2n+2n-2+1}}{(\frac{x^2}{2})^m} = \lim_{x \to 0} \frac{2nx^{4n-1}}{\frac{1}{2^m}x^{2m}} = \lim_{x \to 0} 2^{m+1} n x^{4n-2m-1}$ ⟶ حاصل حد عدد غیر متناهی غیر صفر ← توان صفر

$\begin{cases} 4n - 2m - 1 = 0 \to n = \frac{2m+1}{4} \\ 2^{m+1} n = 32\sqrt{2} \end{cases} \to 2^{m+1} \times \frac{2m+1}{4} = 32\sqrt{2} \to (2m+1)2^{m+1} = 2^7\sqrt{2} \to m = \frac{7}{2}$ $2m + n = 9$

$n = 2$

(۸)

$f(x) = \frac{\sqrt{x}+1}{\sqrt{x}-1} \to f^{-1}(x) \to x = \frac{\sqrt{y}+1}{\sqrt{y}-1} \to y = \left(\frac{x+1}{x-1}\right)^2$

$(F^{-1}(x))' = 2\left(\frac{x+1}{x-1}\right)\left(\frac{-2}{(x-1)^2}\right) \xrightarrow{x=2} 2 \times \frac{3}{1} \times (-2) = -12$ ✓

(۲)

(۹)

$f(x) = (x^2+1)^3(ax+1) \xrightarrow{x=-1} 8(-a+1)$
$\xrightarrow{x=0} 1$

$\frac{1 - 8(a+1)}{0-(-1)} = -11$ ($1 + 8a - 8$, 1)

$8a - 7 = -11$

$8a = -4 \to a = \frac{-1}{2}$

(۲)

$f'(x) = 3 \times 2x(x^2+1)^2(-\frac{1}{2}x+1) + (x^2+1)^3(-\frac{1}{2})$

$x = -2a$ ($-\frac{1}{2}$) (1) $\xrightarrow{x=1} 6 \times 4 \times \frac{1}{2} + 8 \times (-\frac{1}{2}) = 12 - 4 = 8$ ✓

$y = \sin x \cos 2x$

$\hookrightarrow 1 \times (-1) = -1$

$x = \frac{\pi}{2}$

$\frac{\sqrt{2}}{2} \times 0$

$x = \frac{\pi}{4}$

$\frac{-1}{\frac{\pi}{2}} = -\frac{2}{\pi}$

$y = \sin^2 x - \cos^2 x$

$\sin^2 x - \cos^2 x$

$-\cos 2x$

$\xrightarrow{x = \frac{\pi}{2}} -(-1) = 1$

$x = \frac{\pi}{4} \rightarrow 0$

$\frac{1}{\frac{\pi}{2}} = \frac{2}{\pi}$

$\frac{-2}{\frac{2}{\pi}} = -1$

(1.)

(2)