SUBJECT

Year: Month: Day: Page: ( )

تکلیف ۲۳: ۱۸/۵ به نام خدا یکتا دهقانی / دوازدهم ۸

①

$g'(x) = (2\times(\tan^2 x+1)\times \tan x) + (-\sqrt{2}\sin x)\times f'(\tan^2 x+\sqrt{2}\cos x)$

$g'(\frac{\pi}{4}) = (2\times(1+1)\times 1) + (-\sqrt{2}\times\frac{\sqrt{2}}{2})\times f'(2) = 3\times f'(2) = \sqrt{3}$

$f'(2) = \frac{1}{\sqrt{3}} \to 2.$ ✓ ②

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②

$f'(\frac{7\pi}{6}) - 2g'(\frac{7\pi}{6}) = (f-2g)'(\frac{7\pi}{6})$

$(f-2g)(x) \to \frac{8+\cos^3 x}{4-\cos^2 x} - \frac{4}{2-\cos x} = \frac{8+\cos^3 x - 8 - 4\cos x}{4-\cos^2 x}$

$= \frac{\cos x(\cos^2 x - 4)}{4-\cos^2 x} \xrightarrow{\text{ساده کردن}} (f-2g)(x) = -\cos x$ ②

$\to (f-2g)'(x) = -(-\sin x) = \sin x \xrightarrow{x=\frac{7\pi}{6}} -\frac{1}{2} \to 2.$ ✓

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③

$x = \frac{3}{4}^+ \to (4x-3)\sqrt{ax} = y \to y'_1 = 4(\sqrt{ax}) + (4x-3)(\frac{a}{2\sqrt{ax}})$

$x = \frac{3}{4}^- \to (3-4x)\sqrt{ax} = y \to y'_2 = -4(\sqrt{ax}) + (3-4x)(\frac{a}{2\sqrt{ax}})$

$\Rightarrow y'_1 - y'_2 = 8\sqrt{ax} + (8x-6)(\frac{a}{2\sqrt{ax}}) = 2\sqrt{6}$ ①

$\to \frac{8ax + 4ax - 3a}{\sqrt{ax}} = \frac{12ax - 3a}{\sqrt{ax}} \xrightarrow{x=\frac{3}{4}} \frac{9a - \frac{9}{4}a}{\sqrt{\frac{3}{4}a}}$

$\to \frac{27a}{2\sqrt{3a}} = 2\sqrt{6} \to 27\sqrt{a} = 4\sqrt{18} = 12\sqrt{2}$

$\to \sqrt{a} = \frac{12\sqrt{2}}{27} \Rightarrow a = \frac{16\times 2}{81} = \frac{32}{81} \to 2.$

$x \to (\frac{3}{4})^+ \to f(x) = (4x-3)\sqrt{ax} \to f'(x) = 4\sqrt{ax}$ مشتق عامل صفرشونده

$\to f'_+(\frac{3}{4}) = 4\sqrt{\frac{3}{4}a}$

$x \to (\frac{3}{4})^- \to f(x) = (3-4x)\sqrt{ax} \to f'(x) = -4\sqrt{ax}$ مشتق عامل صفر شونده

$f'_-(\frac{3}{4}) = -4\sqrt{\frac{3}{4}a}$

$\to f'_+(\frac{3}{4}) - f'_-(\frac{3}{4}) = 8\sqrt{\frac{3}{4}a} = 2\sqrt{6} \to \sqrt{\frac{3}{4}a} = \frac{\sqrt{6}}{4} \to \frac{3}{4}a = \frac{6}{16} \to a = \frac{1}{2}$

Senobar

(٤) $y = -ax + ٢ \rightarrow$ شیب $= -a = f'(٤)$ — ۱٫۵

$f(٤) = -٤a + ٢$

$f(٤) + f'(٤) = -٤a + ٢ - a = -1 \Rightarrow -۵a + ٢ = +1 \Rightarrow a = \frac{1}{۵}$ — $a = \frac{٣}{۵}$

$f'(٤) = -a = \left(-\frac{1}{۵}\right) \rightarrow ٢$ — $f'(٤) = \frac{-٣}{۵}$

(ز) $٧y - x = ۵ \rightarrow y = \frac{1}{٧}x + \frac{۵}{٧} \rightarrow$ شیب $= \frac{1}{٧}$

$y' = \frac{a(٣x+1) - ٣(ax-1)}{(٣x+1)^٢} = \frac{٣ax + a - ٣ax + ٣}{(٣x+1)^٢} = \frac{a+٣}{(٣x+1)^٢}$

1 — $\Delta = 0 \rightarrow (1٤ - ٧a)^٢ - ٤(٣)(1٢) = 0 \rightarrow \begin{cases} a = \frac{٤}{٧} \\ a = ٤ \end{cases}$

$y = \frac{1}{٧}x + \frac{۵}{٧} = \frac{ax-1}{٣x+1} \rightarrow \frac{٣x^٢}{٧} + \frac{1٤}{٧}x + \frac{۵}{٧} = ax - 1$

$\rightarrow \frac{٣}{٧}x^٢ + \left(\frac{1٤}{٧} - a\right)x + \frac{1٢}{٧} = 0 \rightarrow \Delta = 0$ چون مماس است!

$\left(\frac{1٤}{٧} - a\right)^٢ - \frac{1٤٤}{٤٩} = 0 \rightarrow \frac{1٤}{٧} - a = \pm\frac{1٢}{٧}$

$\rightarrow a = \frac{٤}{٧} \rightarrow \frac{٣}{٧}x^٢ + \frac{1٢}{٧}x + \frac{1٢}{٧} = 0 \rightarrow x = -٢ \rightarrow$ ق ق $\rightarrow m = \frac{1}{٧}$

$\rightarrow a = \frac{٢٨}{٧} \rightarrow \frac{٣}{٧}x^٢ - \frac{1٢}{٧}x + \frac{1٢}{٧} = 0 \rightarrow x = ٢ \rightarrow$ ق ق $\rightarrow m = \frac{1}{٧}$

$٣x^٢ + (1٤ - ٧a)x + 1٢ \xrightarrow{\text{مضاعف}} x = \frac{-b}{٢a} = \frac{-1٤ + ٧a}{٦} \rightarrow \begin{cases} a = ٤ \rightarrow x = ٢ \quad ✓ \\ a = \frac{٤}{٧} \rightarrow x = -٢ \quad ✗ \end{cases}$

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(٢) $y = 0 \rightarrow y' = 0 \Rightarrow f(٢) = 0 = ٨ + ٢a - b \rightarrow ٢a - b = -٨$

$f'(x) = ٣x^٢ + a \rightarrow f'(٢) = 1٢ + a = 0 \rightarrow a = -1٢, \; b = -1٦$

$\Rightarrow b - a = -1٦ - (-1٢) = (-٤) \rightarrow ٢$ — ٢ ✓

CamScanner

$$\begin{cases} 3m - 2m - 1 = 0 \longrightarrow n = \frac{2m+1}{4} \\ 2^{m+1} n = 32\sqrt{2} \end{cases} \longrightarrow 2^{m+1} \times \frac{2m+1}{4} = 32\sqrt{2} \rightarrow (2m+1)2^{m+1} = 2^{7}\sqrt{2} \longrightarrow m = \frac{5}{2}, \quad n = 4$$

$2m+n = 9$

SUBJECT:

Year: Month: Day: Page : ( )

$f'(x) = n \times (2x \times \cos x^2) \times \sin^{1-n}(x^2)$ (1)

$\lim_{x \to 0} \frac{f(x) f'(x)}{(1-\cos x)^m} \longrightarrow \lim_{x \to 0} \frac{2xn \cos x^2 \times \sin^{1-n}(x^2) \times \sin^n(x^2)}{(1-\cos x)^m}$ (1,5)

هم ارزی $\longrightarrow \lim_{x \to 0} \frac{2xn \times x^2 \times x^{2-2n} \times x^{2n}}{\left(\frac{x^2}{2}\right)^m} = \lim_{x \to 0} \frac{2x^5 \times n \times 2^m}{x^{2m}} = 32\sqrt{2}$

$m = \frac{5}{2} \longleftarrow 2m = 5 \longleftarrow$ برای اینکه حد عدد شود

$2 \times n \times \sqrt{32} = 32\sqrt{2} \longrightarrow n = 4 \longrightarrow 2m + n = 5 + 4 = 9$

$f'(x) = \frac{\frac{1}{2\sqrt{x}}(\sqrt{x}-1) - \frac{1}{2\sqrt{x}}(\sqrt{x}+1)}{(\sqrt{x}-1)^2}$ (1) (0,5)

$\rightarrow f'(x) = \frac{\frac{1}{2\sqrt{x}}(\sqrt{x}-1-\sqrt{x}-1)}{(\sqrt{x}-1)^2} = \frac{-1}{\sqrt{x}(\sqrt{x}-1)^2}$

$\rightarrow f(2) = \frac{1}{\sqrt{2}(\sqrt{2}-1)^2} = \frac{1}{\sqrt{2}(2+1-2\sqrt{2})} = \frac{1}{3\sqrt{2}-4}$ ✓

$\frac{f(0) - f(-1)}{0-(-1)} = -11 \rightarrow \frac{1 - (8 \times (-a+1))}{1} = -11$ (9)

$\rightarrow 1 + 8a - 8 = -11 \rightarrow 8a - 7 = -11 \rightarrow 8a = -4 \rightarrow a = -\frac{1}{2}$

$x = -2a = -2 \times -\frac{1}{2} = 1$

$f'(x) = 3(2x)(x^2+1)^2(ax+1) + a(x^2+1)^3$ (2)

$\rightarrow f'(1) = 3(2)(2)^2\left(\frac{1}{2}\right) + (-4) = 12 - 4 = 8$ ✓

Senobar

سوال ۱۶

$f(x) = \frac{\sqrt{x}+1}{\sqrt{x}-1} \rightarrow \sqrt{x}y - y = \sqrt{x}+1 \longrightarrow \sqrt{x}(y-1) = y+1 \rightarrow \sqrt{x} = \frac{y+1}{y-1} \rightarrow x = \left(\frac{y+1}{y-1}\right)^2 \rightarrow y = \left(\frac{x+1}{x-1}\right)^2$

$f^{-1}(x) = \left(\frac{x+1}{x-1}\right)^2 \rightarrow (f^{-1})' = 2\left(\frac{x+1}{x-1}\right) \times \frac{-2}{(x-1)^2} \longrightarrow (f^{-1}(2))' = 2 \times 3 \times (-2) = -12$

CamScanner

$$\begin{cases} g(x) = \sin^2 x - \cos^2 x \\ f(x) = \sin x \cos x \end{cases}$$

Page : ( )

SUBJECT:

Year: Month: Day:

$$\frac{f(\frac{\pi}{4}) - f(\frac{\pi}{2})}{\frac{\pi}{4} - \frac{\pi}{2}} = \frac{-1 - 0}{\frac{\pi}{2}}$$

$$\frac{g(\frac{\pi}{4}) - g(\frac{\pi}{2})}{\frac{\pi}{4} - \frac{\pi}{2}} = \frac{1 - 0}{\frac{\pi}{2}}$$

$\rightarrow \frac{-1}{1} = (-1)$ ✓ (۱)

(۱۰)