14 روشنا ابوسی

۱)

$$g'(x) = 2(\tan x)(1+\tan^2 x) - \sqrt{2}\sin x \times f'(\tan^2 x + \sqrt{2}\cos x)$$

$$g'(\frac{\pi}{4}) = 3\times f'(2) = \sqrt{3} \Rightarrow f'(2) = \frac{\sqrt{3}}{3}$$

۴

۲)

$$f(x) = \frac{(2+\cos x)(4-2\cos x+\cos^2 x)}{(2+\cos x)(2-\cos x)} \qquad g(x) = \frac{2}{2-\cos x}$$

$$f'(\frac{7\pi}{6}) - 2g'(\frac{7\pi}{6}) = (f-2g)'(\frac{7\pi}{6})$$

$$f-2g = \frac{\cos^2 x - 2\cos x + 4 - 4}{2-\cos x} = -\cos x \xrightarrow{\text{مشتق}} = \sin x$$

۴

$$(f-2g)'(\frac{7\pi}{6}) = \sin\frac{7\pi}{6} = -\frac{1}{2}$$

۳)

$$f(x)\begin{cases} x > \frac{3}{4} \to (4x-3)\sqrt{ax} \to f'(x) = 4\sqrt{ax} + (4x-3)\frac{a}{2\sqrt{ax}} \\ \qquad = 4\sqrt{ax} \quad (1) \\ x < \frac{3}{4} \to -(4x-3)\sqrt{ax} \to f'(x) = -(4\sqrt{ax} + (4x-3)\frac{a}{2\sqrt{ax}}) \\ \qquad = -4\sqrt{ax} \quad (2) \end{cases}$$

$$(1)-(2) = 2(4\sqrt{ax}) = 2\sqrt{6} \xrightarrow{x=\frac{3}{4}} 4\sqrt{\frac{3}{4}a} = \sqrt{6} \Rightarrow a = \frac{1}{2}$$

۴

۴)

$$\text{خط} \longrightarrow y = 2 - ax \qquad f(4) = 2-4a \qquad f'(4) = -a$$

$$\Rightarrow f(4)+f'(4) = 2-4a-a = 2-5a = -1 \Rightarrow a = \frac{3}{5}$$

$$\Rightarrow f'(4) = -\frac{3}{5}$$

۴

روشنا الیوسی

۵

$$\frac{1}{7}=\frac{a+3}{(3x+1)^2} \Rightarrow a=\frac{(3x+1)^3-21}{7}$$

۲

$$\frac{5+x}{7}=\frac{ax-1}{3x+1} \Rightarrow \frac{5+x}{7}=\frac{x(3x+1)^2-21x-7}{7(3x+1)}$$

$$\Rightarrow 9x^3+6x^2-20x-7=3x^2+16x+5 \Rightarrow 9x^3+3x^2-36x-12=0$$

$$\Rightarrow x^3(3x+1)-4(3x+1)=(x^2-4)(3x+1)=0$$

$$\xrightarrow{\text{ناحیه اول}} x=2,\ \boxed{a=4}$$

۶ در نقطه (۲,۰) مقادیر مشتق ها برابر

۲

$$f'(x)=3x^2+a \xrightarrow{(2,0)} 12+a=0 \Rightarrow a=-12$$

$$f(x)=x^3-12x-b \xrightarrow{(2,0)} 8-24-b=0 \Rightarrow b=-16 \qquad \Rightarrow b-a=\boxed{-4}$$

۷

۱

$$f'(0)=\lim_{x\to 0}\frac{f(x)-f(0)}{x-0}=\frac{\sin^n(x^2)}{x}\xrightarrow{\text{هم‌ارزی}}=\frac{x^{2n}}{x}=x^{2n-1}$$

$$\lim\frac{f(x)f'(x)}{(1-\cos x)^m}=32\sqrt{2}=\frac{\sin^n(x^2)(x^{2n-1})}{(1-\cos x)^m}$$

$$\xrightarrow{\text{هم‌ارزی}}\frac{x^{2n}\times x^{2n-1}}{\left(\frac{x^2}{2}\right)^m}=\frac{2n\, x^{4n-1}}{(x^{2m})(2^{-m})}=(x^{4n-2m-1})(2^m)=32\sqrt{2}$$

$$\xrightarrow{x=0} 2^{m+1}n=32\sqrt{2} \Rightarrow 2^m=2^{\frac{11}{2}} \Rightarrow m=\frac{11}{2},\ 4n-1=2m=11$$

$$\Rightarrow 4n=12 \Rightarrow n=3 \qquad 2m+n=11+3=\boxed{14}$$

$$\begin{cases}4n-2m-1=0 \longrightarrow n=\frac{2m+1}{4}\\ 2^{m+1}n=32\sqrt{2}\end{cases} \longrightarrow 2^{m+1}\times\frac{2m+1}{4}=32\sqrt{2} \to (2m+1)2^{m+1}=2^7\sqrt{2} \to m=\frac{7}{2},\ n=2 \qquad 2m+n=9$$

روشنا ایوبی

۸

$$y=\frac{\sqrt{x}+1}{\sqrt{x}-1} \Rightarrow f^{-1} \rightarrow x=\frac{\sqrt{y}+1}{\sqrt{y}-1} \Rightarrow y=\left(\frac{x+1}{x-1}\right)^2$$

۲

$$\left(f^{-1}(x)\right)'=\left(\left(\frac{x+1}{x-1}\right)^2\right)'=2\left(\frac{x+1}{x-1}\right)\left(\frac{-2}{(x-1)^2}\right)\xrightarrow{x=2} 2\times\frac{3}{1}(-2)$$

$$=\boxed{-12}$$

۹

$$\frac{f(0)-f(-1)}{0-(-1)}=1-8(-a+1)=8a-7=-11 \Rightarrow a=-\frac{1}{2}$$

۲

$$f'(1)=3(x^2+1)^2(2x)\left(-\frac{x}{2}+1\right)+\left(-\frac{1}{2}\right)(x^2+1)^3=12+(-4)$$

$$=\boxed{8}$$

۱۰

$$y=\sin x\cos 2x$$

$$\lim\frac{f(\frac{\pi}{2})-f(\frac{\pi}{4})}{\frac{\pi}{2}-\frac{\pi}{4}}=\frac{-1-0}{\frac{\pi}{4}}=-\frac{4}{\pi}$$

$$y=\sin^4 x-\cos^4 x=(\sin^2 x-\cos^2 x)\overbrace{(\sin^2 x+\cos^2 x)}^{=1}$$

۲

$$\lim\frac{f(\frac{\pi}{2})-f(\frac{\pi}{4})}{\frac{\pi}{2}-\frac{\pi}{4}}=\frac{1-0}{\frac{\pi}{4}}=\frac{4}{\pi}$$

$$\Rightarrow \frac{-\frac{4}{\pi}}{\frac{4}{\pi}}=\boxed{-1}$$