رزین شهریاری — 14

①

$g(x) = f(\tan^2 x + \sqrt{2}\cos x)$

$g'(x) = (2\tan x(1+\tan^2 x) - \sqrt{2}\sin x)\, f'(\tan^2 x + \sqrt{2}\cos x)$

$\longrightarrow x = \frac{\pi}{4} \quad \left(2(1+1) - \frac{\sqrt{2}\times\sqrt{2}}{2}\right) f'\left(1 + \frac{\sqrt{2}\times\sqrt{2}}{2}\right) = \sqrt{3}$ 2

$(4-1)f'(2) = \sqrt{3} \longrightarrow f'(2) = \frac{\sqrt{3}}{3}$ ✓

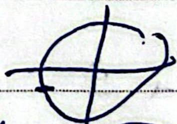
②

$$f(x) = \frac{(\cancel{2+\cos x})(\cos^2 x + 2\cos x + \cancel{4})}{(2-\cos x)(\cancel{2+\cos x})} - \frac{\cancel{4}}{2-\cos x}$$

$$f(x) - 2g(x) = \frac{\cos x(\cos x + 2)}{2-\cos x} \Rightarrow f(x) - 2g(x) = -\cos x$$

2

$f'(x) - 2g'(x) = \sin x$



$\longrightarrow x = \frac{7\pi}{4} \longrightarrow f'(x) - 2g'(x) = -\frac{1}{2}$ ✓

۳) $f(x)=|4x-3|\sqrt{ax} \quad x=\frac{3}{4}$

$+\frac{3}{4}\ \lim_{x\to\frac{3}{4}^+} \frac{(4x-3)\sqrt{ax}-0}{x-\frac{3}{4}} = 4\sqrt{ax}$

$-\frac{3}{4}\ \lim_{x\to\frac{3}{4}^-} \frac{(3-4x)\sqrt{ax}}{x-\frac{3}{4}} = -(4\sqrt{ax})$

۲

$4\sqrt{ax}-(-4\sqrt{ax}) = 8\sqrt{ax} = 8\sqrt{4}$

$4 = \sqrt{\frac{4\cdot 4}{3\cdot a}} \longrightarrow \boxed{a=\frac{1}{3}}$

۴) $y=-ax+2 \qquad A\left|\begin{array}{l}4\\-4a+2\end{array}\right.$

$y'=-a$

$(-a)+(-4a+2)=-1 \qquad -5a+2=-1$

$-5a=-3 \quad a=\frac{3}{5} \longrightarrow \boxed{f'(4)=-\frac{3}{5}}$

۲

$\to 7ax-7=12x^2+x+15x+5 \to 12x^2+(16-7a)x+12=0 \xrightarrow{\Delta=0} (16-7a)^2-4(12)(12)=0 \to \begin{cases} a=\frac{4}{7} \\ a=4 \end{cases}$

۵) $y=\frac{1}{7}x+\frac{5}{7}$

۰/۵

$\frac{1}{7}x+\frac{5}{7}=\frac{ax-1}{3x+1} \longrightarrow 7(ax-1)=x(3x+1)+5(3x+1)$

شیب $\xrightarrow{\text{هم برابر}}$ $\frac{1}{7}=\frac{a+3}{3x+1} \qquad 3x+1=7a+21$

$7=7a+21$

$-14=7a$

$\frac{x+5}{7}=\frac{ax-1}{7(a+3)}$ جایگذاری $\boxed{a=-2}$

Arman

$(a+3)x+5=ax-1$

$3x+5=-1 \ \boxed{x=-2}$

$12x^2+(16-7a)x+12 \xrightarrow{\text{ریشه}} x=\frac{-b}{2a}=\frac{-16+7a}{4} \to \begin{cases} a=4 \to x=3 & ✓ \\ a=\frac{4}{7} \to x=-3 & ✗ \end{cases}$

④ $f(x) = x^3 + ax - b \xrightarrow{\text{[illegible]}}$ [illegible]

۴

$f'(x) = 3x^2 + a \xrightarrow[x=2]{} f'(2) = 12 + a = 0 \longrightarrow a = -12$

$f(x) = x^3 - 12x - b \xrightarrow{x=2} f(2) = 8 - 24 - b \quad b = -16$

$-16 - (-12) = -4$

(جواب پایین صفحه)

⑦ $\cancel{\sin^n(x^2) \times 2x \times \cos}$

$$\lim \frac{\sin^n(x^2) \times n \sin^{n-1}(x^2) \times 2x \times \cos(x^2)}{(1-\cos x)^m}$$

[illegible]

$$\xrightarrow{\text{هم‌ارزی}} \lim \frac{x^{2n} \times n \times (x^2)^{n-1} \times 2x \times 1}{\frac{x^{2m}}{2}} = 4 \times n = 32\sqrt{2}$$

$n = 8\sqrt{2}$

$x^{2n+2n-2+1} = x^{2m}$

$4n - 1 = 2m \quad , \quad 32\sqrt{2} - 1 = 2m$

$32\sqrt{2} - 1 + 8\sqrt{2} = \boxed{40\sqrt{2} - 1}$

(۸) $f(x) = \frac{\sqrt{x}+1}{\sqrt{x}-1}$

$f(x)=\frac{\sqrt{x}+1}{\sqrt{x}-1} \rightarrow \sqrt{x}y - y = \sqrt{x}+1 \longrightarrow \sqrt{x}(y-1)=y+1 \rightarrow \sqrt{x} = \frac{y+1}{y-1} \rightarrow x = \left(\frac{y+1}{y-1}\right)^{۲} \longrightarrow y = \left(\frac{x+1}{x-1}\right)^{۲}$

$f^{-1}\Big|^{۲}_{?} \qquad f\Big|^{?}_{۲} \qquad \frac{\sqrt{x}+1}{\sqrt{x}-1} = ۲$

$f^{-1}(x) = \left(\frac{x+1}{x-1}\right)^{۲} \rightarrow (f^{-1})' = ۲\left(\frac{x+1}{x-1}\right) \times \frac{-۲}{(x-1)^{۲}} \longrightarrow (f^{-1}(۲))' = ۲\times ۳ \times (-۲) = -۱۲$

$\cancel{\sqrt{x}+1 = ۲\sqrt{x}-۲} \quad \cancel{x=۹} \longrightarrow \frac{\left(1+\frac{۲}{۲\sqrt{x}}\right)\times ۸ - (۱)(۱۵)}{۶۴}$

$\cancel{\frac{\frac{۴}{۳}\times ۸ - ۱۵}{۶۴}} = \cancel{\frac{\frac{۳۲}{۳} - ۱۵}{۶۴}} =$

(۲)

$$f'(x) = \frac{\left(\frac{1}{۲\sqrt{x}}\right)(\sqrt{x}-1) - \left(\frac{1}{۲\sqrt{x}}\right)(\sqrt{x}+1)}{(\sqrt{x}-1)^{۲}}$$

$$f'(۹) = \frac{\left(\frac{۱}{۶}\right)(۲) - \left(\frac{۱}{۶}\right)(۴)}{۴} = \frac{\frac{۱}{۳}+\frac{۲}{۳}}{۴} = \frac{۱}{۴}$$

$(f^{-1}(۹))' = ۴$

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(۹) $x=۰ \longrightarrow f(۰) = ۱$

$x=-۱ \longrightarrow f(-۱) = ۸(-a+۱) = -۸a+۸$

$\frac{-۸a+۸-۱}{-۱-۰} = -۱۱ \qquad -۸a+۷ = ۱۱$

(۲)

$-۸a = ۴ \qquad a = -\frac{۱}{۲} \longrightarrow \boxed{a = -\frac{۱}{۲}}$

$f'(x) = ۳(x^{۲}+۱)^{۲}(۲x)\left(-\frac{۱}{۲}x+۱\right) + \left(-\frac{۱}{۲}\right)(x^{۲}+۱)^{۳}$

$x=۱ \rightarrow ۳(۴)(۲)\left(\frac{۱}{۲}\right) + \left(-\frac{۱}{۲}\right)(۸)$

$f'(۱) = ۱۲ - ۴ = ۸$

سوال ۱۷

$$\lim_{x\to 0} \frac{\sin^n(x^2) \times n\sin^{n-1}(x^2) \times \cos(x^2) \times (2x)}{\left(2\sin^2\frac{x}{2}\right)^m} \xrightarrow{\sin\approx} \lim_{x\to 0} \frac{(x^2)^n\, n (x^2)^{n-1} \times 2x}{\left(2\left(\frac{x}{2}\right)^2\right)^m} =$$

$$\lim_{x\to 0} \frac{2n x^{2n+2n-2+1}}{\left(\frac{x^2}{2}\right)^m} = \lim_{x\to 0} \frac{2n x^{4n-1}}{\frac{1}{2^m} x^{2m}} = \lim_{x\to 0} 2^{m+1} n\, x^{4n-2m-1}$$

حاصل حد عدد غیر متناهی غیر صفر ← توان x صفر باشد

$$\begin{cases} 4n-2m-1=0 \longrightarrow n=\frac{2m+1}{4} \\ 2^{m+1} n = 32\sqrt{2} \end{cases} \longrightarrow 2^{m+1} \times \frac{2m+1}{4} = 32\sqrt{2} \to (2m+1)2^{m+1} = 2^7\sqrt{2} \longrightarrow m=\frac{7}{2}, \quad n=2$$

$$2m+n=9$$

سوال ۱۰

$$\begin{cases} y_1\left(\frac{\pi}{4}\right) = \sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{2}\right) = 0 \\ y_1\left(\frac{\pi}{2}\right) = \sin\left(\frac{\pi}{2}\right)\cos(\pi) = -1 \end{cases} \longrightarrow \frac{\Delta y_1}{\Delta x} = \frac{-1-0}{\frac{\pi}{2}-\frac{\pi}{4}} = \frac{-4}{\pi}$$

$$y_2 = \sin^4 x - \cos^4 x = -\cos 2x$$

$$\begin{cases} y_2\left(\frac{\pi}{4}\right) = -\cos\left(\frac{\pi}{2}\right) = 0 \\ y_2\left(\frac{\pi}{2}\right) = -\cos(\pi) = 1 \end{cases} \longrightarrow \frac{\Delta y_2}{\Delta x} = \frac{1-0}{\frac{\pi}{2}-\frac{\pi}{4}} = \frac{4}{\pi}$$

$$\longrightarrow \frac{\frac{\Delta y_1}{\Delta x}}{\frac{\Delta y_2}{\Delta x}} = \frac{-\frac{4}{\pi}}{\frac{4}{\pi}} = -1$$