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طه سلطان الکسبی

(۱۵)

$\Delta=0 \Rightarrow y_1=\frac{ax-1}{3x+1},\quad y_2=\frac{5+x}{7} \Rightarrow \frac{ax-1}{3x+1}=\frac{5+x}{7} \Rightarrow 7ax-7=3x^2+16x+5$

$\Rightarrow 3x^2+(16-7a)x+12=0$

$\Delta=0 \Rightarrow (16-7a)^2-4(3)(12)=0$

$\Rightarrow (16-7a)^2=144$

$\Rightarrow 16-7a=12 \Rightarrow a=\frac{4}{7}$ ✗

$16-7a=-12 \Rightarrow a=4$ ✓

$a=4 \Rightarrow 3x^2-12x+12=0$

$\Rightarrow 3(x-2)^2=0 \Rightarrow x=2$ ✓

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(۱۰)

$$\frac{\dfrac{f(\frac{\pi}{2})-f(\frac{\pi}{4})}{\frac{\pi}{2}-\frac{\pi}{4}}}{\dfrac{g(\frac{\pi}{2})-g(\frac{\pi}{4})}{\frac{\pi}{2}-\frac{\pi}{4}}}=\frac{\underbrace{\sin\frac{\pi}{2}\times\cos\pi}_{-1}-\underbrace{\sin\frac{\pi}{4}\times\cos\frac{\pi}{2}}_{0}}{(\sin^4\frac{\pi}{2}-\cos^4\frac{\pi}{2})-(\sin^4\frac{\pi}{4}-\cos^4\frac{\pi}{4})}=-1$$

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$$\frac{f(0)-f(-1)}{0-(-1)}=\frac{1-8(1-a)}{1}=8a-7=-11 \Rightarrow 8a=-4 \Rightarrow a=-\frac{1}{2}$$

$f'(x)=3(x^2+1)^3(-\frac{1}{2}x+1)+(-\frac{1}{2})(x^2+1)^3 \Rightarrow f'(-1)=12-4=8$

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$x>\frac{3}{4} \Rightarrow f(x)=(4x-3)\sqrt{ax} \Rightarrow f'(x)=4\sqrt{ax}$

$f'_+(\frac{3}{4})=4\sqrt{\frac{3}{4}a}=4\frac{\sqrt{3a}}{2}=2\sqrt{3a}$

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$x<\frac{3}{4} \Rightarrow f(x)=-(4x-3)\sqrt{ax} \Rightarrow f'(x)=-4\sqrt{ax}$ $\Rightarrow a=\frac{1}{2}$

$\Rightarrow f'_-(\frac{3}{4})=-4\sqrt{\frac{3}{4}a}=-4\frac{\sqrt{3a}}{2}=-2\sqrt{3a}$

$f'_+(\frac{3}{4})-f'_-(\frac{3}{4})=2\sqrt{6} \Rightarrow 2\sqrt{3a}+2\sqrt{3a}=2\sqrt{6} \Rightarrow (4\sqrt{3a})^2=(2\sqrt{6})^2 \quad 48a=24$

NEGARNOTE

$y=\frac{\sqrt{x}+1}{\sqrt{x}-1} \Rightarrow \sqrt{x}\,y - y = \sqrt{x}+1 \Rightarrow \sqrt{x}\cdot y - \sqrt{x} = y+1$ 4 1

$\Rightarrow \sqrt{x}(y-1) = y+1 \Rightarrow \sqrt{x} = \frac{y+1}{y-1} \Rightarrow x = \left(\frac{y+1}{y-1}\right)^2 \Rightarrow f^{-1}(x) = \left(\frac{x+1}{x-1}\right)^2$

$\longrightarrow f^{-1'}(x) = 2\left(\frac{x+1}{x-1}\right)^1 \left(\frac{1(x-1)-1(x+1)}{(x-1)^2}\right) \Rightarrow x = 6 \times (-2) = -12$

$\cancel{f(x) = 2x\,x\cos^2 \sin x^{-1}x^{2} \; \lim \frac{f(x)\times f'(x)}{(1+\cos x)^x}}$

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$\lim_{x\to 0} \frac{x^{2x} \; 2\times x^{2x-1}}{\left(\frac{x^2}{2}\right)^x} = \lim_{x\to 0} x\,2^{x+1}\,\frac{x^{4x-1}}{x^{2x}} = 2^{\frac{11}{2}}$

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$4x-1 = 2x \longrightarrow x \times 2^{x+1} = 2^{\frac{11}{2}} \Rightarrow n = 2$

$m = \frac{7}{2}$

$\Rightarrow 2m+n = 7+2 = 9$

سوال ۱۱

$$g'(x)=\left(2\tan x\,(1+\tan^2 x-\sqrt{2}\sin x)\,f'(\tan^2 x+\sqrt{2}\cos x)\right)$$

$$\xrightarrow{x=\pi/4} g'(\tfrac{\pi}{4})=3f'(2)=\sqrt{3}\longrightarrow f'(2)=\frac{1}{\sqrt{3}}\longrightarrow f'(2)=\frac{\sqrt{3}}{3}$$

سوال ۱۲

$$f'(\tfrac{7\pi}{6})-2g'(\tfrac{7\pi}{6})=(f-2g)'(\tfrac{7\pi}{6})$$

$$\rightarrow (f-2g)'=\left(\frac{8+\cos^3 x}{4-\cos^2 x}-\frac{4}{2-\cos x}\right)=\left(\frac{(\cos x+2)(\cos^2 x-2\cos x+4)}{(2-\cos x)(2+\cos x)}-\frac{4}{2-\cos x}\right)=$$

$$\left(\frac{\cos(\cos x-2)}{-(\cos x-2)}\right)'=-(\cos x)'=\sin x\xrightarrow{x=7\pi/6}\sin(\tfrac{7\pi}{6})=\frac{-1}{2}$$

سوال ۱۴

$$y=-ax+2$$

$$f(4)=-4a+2\ ,\ f'(4)=-a$$

$$f(4)+f'(4)=-4a+2-a=-1\longrightarrow a=\frac{3}{5}\longrightarrow f'(4)=-a=-\frac{3}{5}=-0.6$$

سوال ۱۶

$$\begin{cases}f(2)=0\longrightarrow 8+2a-b=0\\ f'(2)=0\longrightarrow 3x^2+a=0\longrightarrow 12+a=0\rightarrow a=-12\end{cases}$$

$$\rightarrow 2a-b=-8\xrightarrow{a=-12} b=-16$$

$$b-a=-16-(-12)=-4$$