(۱

$g'(\frac{\pi}{4}) = \sqrt{3} = f'(\underbrace{\tan^2 x + \sqrt{2}\cos x}_{2}) \times (\underbrace{2\tan x(1+\tan^2 x)}_{4} - \underbrace{\sqrt{2}\sin x}_{1}) \xrightarrow{x=\frac{\pi}{4}}$

$3f'(2) = \sqrt{3} \to f'(2) = \frac{\sqrt{3}}{3}$ ✓ (۲)

(۲ (۱٫۵)

$f'(\frac{7\pi}{4}) - 2g'(\frac{7\pi}{4}) = (f-2g)'(\frac{7\pi}{4}) \to \frac{8+\cos^3 x}{4-\cos x} - \frac{4}{2-\cos x} = \frac{8+\cos^3 x - 4(2+\cos x)}{4-\cos^2 x} =$

$\frac{\cos x(\cos^2 x - 4)}{4-\cos^2 x} = -\cos x = \frac{\sqrt{2}}{2} \xrightarrow{\text{مشتق}} \sin x \xrightarrow{x=7\pi/4} \sin(\frac{7\pi}{4}) = \frac{-1}{2}$

(۳

$\text{مشتق راست} = \frac{(4x-3)a}{2\sqrt{ax}} + 4\sqrt{ax}$ $\qquad \text{مشتق چپ} = \frac{-a(4x-3)}{2\sqrt{ax}} - 4\sqrt{ax}$

$\text{اختلاف مشتق چپ و راست} = \frac{(4x-3)a}{\sqrt{ax}} + 8\sqrt{ax} \to 8\sqrt{\frac{3}{4}a} = 2\sqrt{6} \to a = \frac{1}{2}$ ✓ (۲)

(۴

$f(4) = 2-4a \to 2-5a = -1 \to a = \frac{3}{5}$ $\qquad f'(4) = -a = -\frac{3}{5}$ ✓ (۲)

$f'(4) = -a$

(۵

$\frac{x+5}{7} = \frac{ax-1}{3x+1} \to 7(ax-1) = (x+5)(3x+1) \to$

$3x^2 + (16-7a)x + 12 = 0 \xrightarrow{\Delta=0} (16-7a)^2 = 144 \to 16-7a = \pm 12 \to$ (۲)

$a = \frac{4}{7} \to \frac{x+5}{7} = \frac{\frac{4}{7}x-1}{3x+1} \to x = -2$

$a = 4 \to \frac{x+5}{7} = \frac{4x-1}{3x+1} \to x = 2$

نقطه بجورد در ناحیه اول $\Leftarrow x \geq 0 \to a=4$ قابل قبول ✓

(۶ (۱٫۵)

$8-2a+b=0 \to 2a-b=8$

$f'(a) \times 3x^2 + a \xrightarrow{x=2} 12 + a = 0 \to a = -12$

$\} \to b = -32$

$2a-b = -8 \xrightarrow{a=-12} b = -16$

$b-a = -32-(-12) = -20$

$b-a = -16-(-12) = -4$

(۷ (۲)

$\sin x \simeq x \to x \to 0 \ , \ 1-\cos x \simeq \frac{x^2}{2} \to x \to 0 \to$

$\frac{f(x)f'(x)}{(1-\cos x)^2} = \frac{x^{2n} \times 2n\, x^{2n-1}}{(\frac{x^2}{2})^m} = \frac{x^{4n-1} \times 2n}{2^{-m} \times x^{2m}} = \div = 32\sqrt{2} \to 4n-1 = 2m \to$

$2n \times 2^m = 32\sqrt{2} \to n \times 2^{\frac{4n+1}{2}} = 2^{\frac{11}{2}} \to n=2 \to m = \frac{7}{2} \to 2m+n = 7+2 = 9$ ✓

$$\frac{\sqrt{x}+1}{\sqrt{x}-1} = y \longrightarrow \frac{\sqrt{y}+1}{\sqrt{y}-1} = x \longrightarrow \sqrt{y} = \frac{x+1}{x-1} \longrightarrow y = \frac{(x+1)^2}{(x-1)^2} \longrightarrow$$ (۸

$$y' = \frac{-4x^2+4}{(x-1)^4} \xrightarrow{x=2} y' = \frac{-16+4}{1} = -12$$ ✓

(۲)

$$\frac{1-\lambda(1-a)}{1} = -11 \longrightarrow 1-\lambda+\lambda a = -11 \longrightarrow \lambda a = -4 \longrightarrow a = -\frac{1}{2} \longrightarrow -2a = 1$$ (۹

(۲)

$$f'(x) = -\frac{1}{2}(x^2+1)^2 + 2(x^2+1)^2 \times 2x\left(-\frac{1}{2}x+1\right) \longrightarrow f'(2) = -4+12 = 8$$ ✓

$$\frac{-1}{\frac{\pi}{4}} = \frac{-4}{\pi} \quad , \quad \frac{(1-0)-\left(\frac{1}{2}-\frac{1}{2}\right)}{\frac{\pi}{4}} = \frac{4}{\pi}$$ (۱۰

(۲)

$$\frac{-\frac{4}{\pi}}{\frac{4}{\pi}} = -1$$ ✓