مینا رضایی ۱۹٫۵

۱) $g'(x) = 2(\tan x)(1+\tan^2 x) - \sqrt{2}\sin x \times f'(\tan^2 x + \sqrt{2}\cos x)$

$g'(\frac{\pi}{4}) = 3 \times f'(2) = \sqrt{3} \longrightarrow f'(2) = \frac{\sqrt{3}}{3}$ ✓ ۲

۲) $f(x) = \dfrac{(2+\cos x)(4-2\cos x+\cos^2 x)}{(2+\cos x)(2-\cos x)}$ $\qquad g(x) = \dfrac{2}{2-\cos x}$

$f'(\frac{7\pi}{6}) - 2g'(\frac{7\pi}{9}) = (f-2g)'(\frac{7\pi}{6})$ ۲

$f-2g = \dfrac{\cos^2 x - 2\cos x + 4 - 4}{2-\cos x} = -\cos x \xrightarrow{\text{مشتق}} \sin x$

$(f-2g)'\frac{7\pi}{6} = \sin\frac{7\pi}{6} = -\frac{1}{2}$ ✓

۳)

$$f(x)\begin{cases} x > \frac{3}{4} \to (4x-3)\sqrt{ax} \longrightarrow f'(x) = 4\sqrt{ax} + (4x-3)\frac{a}{2\sqrt{ax}} = 4\sqrt{ax} \\ x < \frac{3}{4} \to -(4x-3)\sqrt{ax} \longrightarrow f'(x) = -\left(4\sqrt{ax} + (4x-3)\frac{a}{2\sqrt{ax}}\right) = -4\sqrt{ax} \end{cases}$$ ۲

$\xrightarrow{\text{اختلاف}} 2 \times 4\sqrt{ax} = 2\sqrt{6} \xrightarrow{x=\frac{3}{4}} 4\sqrt{\frac{3}{4}a} = \sqrt{6} \longrightarrow a = \frac{1}{2}$ ✓

۴) $\xrightarrow{\text{خط}} y = 2 - ax \xrightarrow[\text{برابر}]{\text{مقدار و مشتق}}$ ① $2-4a = f(4)$ ② $-a = f'(4)$

$f(4)+f'(4) = 2-4a-a = 2-5a = -1 \longrightarrow a = \frac{3}{5}$ ۲

$f'(4) = -a \longrightarrow f'(4) = -\frac{3}{5}$ ✓

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۵)

① شیب‌ها برابر $\to \frac{1}{7} = \frac{a+3}{(3x+1)^2} \longrightarrow a = \frac{(3x+1)^2 - 21}{7}$

② مقدارها برابر $\longrightarrow \frac{5+x}{7} = \frac{ax-1}{3x+1} \xrightarrow{\text{جایگذاری } a} \frac{x+5}{7} = \frac{x(3x+1)^2 - 21x - 7}{7(3x+1)}$

$\longrightarrow 9x^3 + 6x^2 - 20x - 7 = 3x^2 + 16x + 5 \longrightarrow 9x^3 + 3x^2 - 36x - 12 = 0$

$\xrightarrow{\div 3} x^2(3x+1) - 4(3x+1) = (x^2-4)(3x+1) = 0$ ۲

تا جواب قبول $\longrightarrow x = 2 \;,\; a = \frac{49-21}{7} = \underline{4}$

۶) در نقطه (۲و۰) مقدار و شیب‌ها برابر

$f'(x) = 3x^2 + a \xrightarrow{(2,0)} 12 + a = 0 \longrightarrow a = -12$ ۲

$f(x) = x^3 - 12x - b \xrightarrow{(2,0)} 8 - 24 - b = 0 \to b = -16$

$\Big\} \; b - a = \boxed{-4}$

۷) $f'(0) = \lim_{x \to 0} \frac{f(x) - \cancel{f(0)}}{x - \cancel{0}} = \frac{\sin^n(x^2)}{x} \xrightarrow{\text{هم‌ارزی}} \frac{x^{2n}}{x} = x^{2n-1}$

$\lim \frac{f(x) f'(x)}{(1-\cos x)^m} = \frac{\sin^n(x^2) \times x^{2n-1}}{(1-\cos x)^m} \xrightarrow{\text{هم‌ارزی}} \frac{x^{2n} \times x^{2n-1}}{\left(\frac{x^2}{2}\right)^m}$ ۱,۵

$= \frac{2n\, x^{4n-1}}{x^{2m} \times 2^{-m}} = n \times x^{4n-2m-1} \times 2^{m+1} = 32\sqrt{2} \xrightarrow{x=0} 2^m = 32\sqrt{2}$

$\longrightarrow m = \frac{11}{2} \quad , \quad 4n - 1 = 2m \longrightarrow 4n = 12 \to n = 3$

ـ باید توان‌های x صورت و مخرج برابر باشند

$2m + n = 14$

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$\begin{cases} 4n - 2m - 1 = 0 \longrightarrow n = \frac{2m+1}{4} \\ 2^{m+1} n = 32\sqrt{2} \end{cases} \longrightarrow 2^{m+1} \times \frac{2m+1}{4} = 32\sqrt{2} \to (2m+1) 2^{m+1} = 2^7\sqrt{2} \longrightarrow m = \frac{7}{2}$ $\quad 2m + n = 9$

$n = 2$

(۸

$$y = \frac{\sqrt{x}+1}{\sqrt{x}-1} \xrightarrow{f^{-1}} x = \frac{\sqrt{y}+1}{\sqrt{y}-1} \longrightarrow y = \left(\frac{x+1}{x-1}\right)^{۲}$$

۲

$$\left(f^{-1}(x)\right)' = \left(\left(\frac{x+1}{x-1}\right)^{۲}\right)' = ۲ \times \frac{x+1}{x-1} \times \frac{-۲}{(x-1)^{۲}} \xrightarrow{x=۲} ۲ \times \frac{۳}{1} \times \frac{-۲}{1} = -۱۲$$

(۹

$$\frac{f(0)-f(-1)}{0-(-1)} = 1 \quad ۸\times(-a+1) = ۸a - ۷ = -۱۱ \longrightarrow a = -\frac{1}{۲}$$

۲

$$f'(1) = ۳\,(x^{۲}+1)^{۲}(۲x)\left(-\frac{1}{۲}x+1\right) + \left(\frac{-1}{۲}\right)(x^{۲}+1)^{۳} = ۱۲ + (-۴) = ۸$$

(۱۰

$$y = \sin x \cos ۲x$$

$$\lim \frac{f(\frac{\pi}{۲}) - f(\frac{\pi}{۴})}{\frac{\pi}{۲} - \frac{\pi}{۴}} = \frac{-1-0}{\frac{\pi}{۴}} = -\frac{۴}{\pi}$$

$$y = \sin^{۴} x - \cos^{۴} x = (\sin^{۲} x - \cos^{۲} x)\underbrace{\cancel{(\sin^{۲} x + \cos^{۲} x)}}_{1}$$

$$\lim \frac{f(\frac{\pi}{۲}) - f(\frac{\pi}{۴})}{\frac{\pi}{۲} - \frac{\pi}{۴}} = \frac{1-0}{\frac{\pi}{۴}} = \frac{۴}{\pi}$$

۲

$$\frac{-\frac{۴}{\pi}}{\frac{۴}{\pi}} = -1$$