اهدا ارشری ۱۹٫۵

$g'(x) = 2(\tan^2 x+1)\times \tan x) + (-\sqrt{2}\sin x)\times f'(\tan^2 x+\sqrt{2}\cos x)$ ①

$g'(\frac{\pi}{4}) = (\underbrace{2\times 2\times 1}_{4}) + (\underbrace{-\sqrt{2}\times\frac{\sqrt{2}}{2}}_{-1/2})\times f'(2)$

$\longrightarrow 3\times f'(2) = \sqrt{3} \longrightarrow f'(2) = \frac{1}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

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②

$f'(\frac{7\pi}{6}) - 2g'(\frac{7\pi}{6}) = ?$

$\longrightarrow (f-2g)'(\frac{7\pi}{6})$ ابتدا تابع را می‌سازیم سپس ساده می‌کنیم و در نهایت مشتق آن را می‌گیریم.

$(f-2g)(x) \rightarrow \frac{8+\cos^3 x}{4-\cos^2 x} - \frac{4}{2-\cos x} = \frac{8+\cos^3 x-8-4\cos x}{4-\cos^2 x}$

$\frac{\cos x(\cos^2 x-4)}{4-\cos^2 x} \rightsquigarrow (f-2g)(x) = -\cos x$ ۲

$(f-2g)'(x) = -1 - \sin x = \sin x \xrightarrow[x=\frac{7\pi}{6}]{\text{اگر}} \boxed{-\frac{1}{2}}$

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③

$\rightarrow (4x-3)\sqrt{ax} \xrightarrow{\text{مشتق}} 4\sqrt{ax} + \frac{a}{2\sqrt{ax}}(\underbrace{4x-3}_{})$

$\rightarrow (-4x+5)\sqrt{ax} \xrightarrow[]{\text{مشتق}} -4\sqrt{ax} + \frac{a}{2\sqrt{ax}}(-4x+3)$ ۲

$4\sqrt{ax}\ \boxed{-}(-4\sqrt{ax}) = 8\sqrt{ax} \rightarrow 4\sqrt{3a}$ $\boxed{a=\frac{1}{2}}$

$\cancel{4}^{2}\sqrt{3a} = \cancel{2}\times\sqrt{9} \longrightarrow \sqrt{3a} = \frac{\sqrt{9}}{2} \longrightarrow 3a = \frac{9}{4} \rightarrow a = \frac{9}{4\times 3}$

۴

$y + ax = 2 \longrightarrow y = -ax + 2$

$f(4) \longrightarrow y = -4a + 2$

$f'(4) \longrightarrow y = -a$

$f(4) + f'(4) = -1 \longrightarrow -4a + 2 - a = -1 \longrightarrow 3 = 5a \longrightarrow a = \frac{6}{10}$

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$f'(4) \longrightarrow y = -a \longrightarrow \boxed{y = -\frac{6}{10}}$

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۵ چون ۲ منحنی بر هم مماس اند پس معادله تلاقی آن ها باید دارای ریشه مضاعف باشد یعنی $\Delta = 0$

$\frac{ax-1}{3x+1} = \frac{5+x}{7} \longrightarrow 7ax - 7 = 3x^2 + 16x + 5$

$3x^2 + (16 - 7a)x + 12 = 0$

$\Delta = 0 \longrightarrow (16 - 7a)^2 - (12 \times 12) = 0$

$(16 - 7a)^2 = 144 \longrightarrow \begin{cases} 16 - 7a = 12 \to a = \frac{4}{7} \\ 16 - 7a = -12 \to a = 4 \end{cases}$

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$a = 4 \longrightarrow 3x^2 - 12x + 12 = 0 \longrightarrow 3(x-2)^2 = 0 \longrightarrow x = 2$

$a = 4$ قابل قبول است

۶

$f(2) = 0 \longrightarrow 2^3 + 2a - b = 0 \longrightarrow 2a - b = -8$

$f'(x) = 3x^2 + a \xrightarrow{x=2} 12 + a = 0 \longrightarrow a = -12$

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$-24 - b = -8 \longrightarrow \boxed{-16 = b}$

$b - a \longrightarrow -16 - (-12) = -4$

۷

$$\lim_{x\to 0}\frac{f(x)\,f'(x)}{(1-\cos x)^m} \rightsquigarrow \lim_{x\to 0}\frac{2x\cos x^2 \times \sin^{1-n}(x^2)\times \sin^2(x^2)}{(1-\cos x)^m}$$

۱٫۵

$$\longrightarrow \frac{2x\, n \times x^2 \times x^{2-2n} \times x^{2n}}{\left(\frac{x^2}{2}\right)^m} \to \lim_{x\to 0}\frac{2x^5 \times n \times 2^m}{x^{2m}} = 32\sqrt{2}$$

$$\lim_{x\to 0}\frac{2nx^{2n+2n-2+1}}{\left(\frac{x^2}{2}\right)^m} = \lim_{x\to 0}\frac{2nx^{4n-1}}{\frac{1}{2^m}x^{2m}} = \lim_{x\to 0} 2^{m+1}\,n\,x^{4n-2m-1} \rightsquigarrow$$ حاصل حد عدد غیر متناهی غیرصفر ← توان صفر شود

$$\boxed{m = \frac{5}{2}}$$

$$\begin{cases} 4n-2m-1=0 \longrightarrow n=\frac{2m+1}{4} \\ 2^{m+1}n = 32\sqrt{2} \end{cases} \longrightarrow 2^{m+1}\times\frac{2m+1}{4} = 32\sqrt{2} \to (2m+1)2^{m+1} = 2^7\sqrt{2} \to m=\frac{7}{2},\ n=2 \qquad 2m+n=9$$

$$2\times n\times 4\sqrt{2} = 32\sqrt{2} \longrightarrow n=4$$

بنتیجه

$$2m+n \longrightarrow \boxed{5+4=9}$$

جواب = ۹

۸

$$f(x) = \frac{\sqrt{x}+1}{\sqrt{x}-1} \to y = \frac{\sqrt{x}+1}{\sqrt{x}-1}$$

معکوس تابع را بدست آوریم

$$y\sqrt{x} - y = \sqrt{x}+1 \longrightarrow y\sqrt{x}-\sqrt{x} = y+1 \to \sqrt{x}(y-1) = y+1$$

$$\longrightarrow \sqrt{x} = \frac{y+1}{y-1} \longrightarrow x = \left(\frac{y+1}{y-1}\right)^2$$

$$\xrightarrow[\text{عوض}]{\text{جابجایی } x,y} y = \left(\frac{x+1}{x-1}\right)^2 \longrightarrow f^{-1}(x) = \left(\frac{x+1}{x-1}\right)^2$$

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$$\xrightarrow{\text{مشتق می‌گیریم}} 2\left(\frac{x+1}{x-1}\right)\left(\frac{\cancel{x}-1-\cancel{x}-1}{(x-1)^2}\right) \to 2\left(\frac{x+1}{x-1}\right)\left(\frac{-2}{(x-1)^2}\right)$$

$$\xrightarrow{x=2} 2\left(\frac{3}{1}\right)\left(\frac{-2}{1}\right) = \boxed{-12}$$ ✓

CamScanner

۹

آهنگ تغییر متوسط

$$\frac{f(0)-f(-1)}{0-(-1)} = \frac{1-8(1-a)}{1} = 8a-7 = -11$$

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$$8a = -4 \longrightarrow a = -\frac{1}{2}$$

آهنگ تغییر لحظه‌ای

$$\longrightarrow f'(x) = 3(x^2+1)^2(2x)\left(-\frac{1}{2}x+1\right) + \left(-\frac{1}{2}\right)(x^2+1)^3$$

$$\longrightarrow f'(1) = 12-4 = 8$$

۱۰

$$f(x) = \sin x \cos 2x$$
$$g(x) = \sin^4 x - \cos^4 x$$

$$\frac{\dfrac{f(\frac{\pi}{2})-f(\frac{\pi}{4})}{\frac{\pi}{2}-\frac{\pi}{4}}}{\dfrac{g(\frac{\pi}{2})-g(\frac{\pi}{4})}{\frac{\pi}{2}-\frac{\pi}{4}}} = \frac{\overbrace{\sin\frac{\pi}{2}\cos\pi}^{-1} - \overbrace{\sin\frac{\pi}{4}\cos\frac{\pi}{2}}^{0}}{\underbrace{\left(\sin^4\frac{\pi}{2}-\cos^4\frac{\pi}{2}\right)}_{1} - \underbrace{\left(\sin^4\frac{\pi}{4}-\cos^4\frac{\pi}{4}\right)}_{0}} = -1$$

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