نام و نام خانوادگی ...ستایش معصوم‌زاده... پاسخنامه تشریحی تکلیف شماره ...۱۲... کلاس ...دوازدهم تجربی A...

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$g'(x) = f'(\tan^2 x + \sqrt{2}\cos x)(1+\tan^2 x - \sqrt{2}\sin x)$

$x=\frac{\pi}{4} \to g'(\frac{\pi}{4}) = f'(1+\sqrt{2}\times\frac{\sqrt{2}}{2})(1+1-\frac{\sqrt{2}\times\sqrt{2}}{2})$

$\sqrt{3} = f'(2)\times 1 \longrightarrow f'(2) = \sqrt{3}$

$g'(\frac{\pi}{4}) = 3f'(2) = \sqrt{3} \longrightarrow f'(2) = \frac{1}{\sqrt{3}} \longrightarrow f'(2) = \frac{\sqrt{3}}{3}$

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$f(x) = \frac{8+\cos^3 x}{4-\cos^2 x} \qquad g(x) = \frac{2}{2-\cos x} \qquad \sin\frac{7\pi}{6} = -\frac{1}{2}$

$\frac{(\cos^2 x - 2\cos x + 4)(\cos x + 2)}{(2-\cos x)(2+\cos x)} \qquad f'(\frac{7\pi}{6}) - 2g'(\frac{7\pi}{6}) = (f-2g)'(\frac{7\pi}{6})$

$f-2g \longrightarrow \frac{\cos^2 x - 2\cos x + 4 - 4}{2-\cos x} = -\cos x \to (f-2g)'(x) = \sin x$

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$x > \frac{3}{4} \to f(x) = (4x-3)\sqrt{ax} \to f'(x) = 4\sqrt{ax} + (4x-3)\frac{a}{2\sqrt{ax}} = 4\sqrt{ax}$

$x < \frac{3}{4} \to f(x) = -(4x-3)\sqrt{ax} \to f'(x) = -(4\sqrt{ax} + (4x-3)\frac{a}{2\sqrt{ax}}) = -4\sqrt{ax}$

$\xrightarrow{\text{حتما}} \cancel{f}(4\sqrt{ax}) = \cancel{4}\sqrt{6} \xrightarrow{x=\frac{3}{4}} 4\sqrt{\frac{3}{4}a} = \sqrt{6} \to 16\times\frac{3}{4}a = 6 \to a = \frac{1}{2}$

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$\xrightarrow{\text{باید}} f(x) = g \quad , \quad f'(x) = g'$

$f(4) + f'(4) = -5a + 2 = -1 \longrightarrow a = \frac{3}{5}$

$f'(4) = -a = -\frac{3}{5}$

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$y = \frac{x}{7} + \frac{5}{7} \to y' = \frac{1}{7}$

$y = \frac{ax-1}{3x+1} \to y' = \frac{a+3}{(3x+1)^2}$

$\frac{a+3}{(3x+1)^2} = \frac{1}{7} \to a = \frac{(3x+1)^2 - 21}{7}$

$\frac{x}{7} + \frac{5}{7} = \frac{ax-1}{3x+1} \to \frac{x+5}{7} = \frac{x(3x+1)^2 - 21x - 7}{7(3x+1)} \to 9x^3 + 6x^2 - 20x - 7 = 3x^2 + 16x + 5$

$\to 9x^3 + 3x^2 - 36x - 12 = 0 \xrightarrow{\text{تجزیه}} x^2(3x+1) - 4(3x+1) = (x^2-4)(3x+1) = 0$

$x = 2 \leftarrow$

$\boxed{a = 4}$

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$f(x)=x^3+ax-b \qquad f'(x)=3x^2+a$

مماس
$x=2 \Rightarrow f(2)=0 \to 8+2a-b=0 \to 2a-b=-8$
$f'(2)=0 \to 12+a=0 \to a=-12$ $\Big\} \to b=-16$

$b-a=-16-(-12)=\boxed{-4}$ ۶

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$f'(0)=\lim_{x\to 0}\dfrac{f(x)-f(0)}{x-0}=\dfrac{\sin^n(x^2)}{x}\xrightarrow{\text{هم‌ارزی}}\dfrac{x^{2n}}{x}=x^{2n-1}$ $\quad n=\frac{7}{2}$ $\quad 2m+n=9$ $\quad n=2$

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$\lim_{x\to 0}\dfrac{f(x)f'(x)}{(1-\cos x)^m}=32\sqrt{2}\longrightarrow\dfrac{\sin^n(x^2)\,x^{2n-1}}{(1-\cos x)^m}=32\sqrt{2}\xrightarrow{\text{هم‌ارزی}}\dfrac{x^n x^{4n-1}}{\frac{x^{2m}}{2^m}}\;\; x^{4n-2m-1}\quad m+1 \;\; \times 2^{\dots}=32\sqrt{2}$ ۷

$\begin{cases}4n-2m-1=0\longrightarrow n=\frac{2m+1}{4}\\ 2^{m+1}n=32\sqrt{2}\end{cases}\longrightarrow$

$2^{m+1}\times\frac{2m+1}{4}=32\sqrt{2}\to(2m+1)2^{m+1}=2^7\sqrt{2}$ $\quad$ شرط
$4n=2m+1$ $\quad 4n=12\Rightarrow n=3$ $\quad 2^m=2^{\frac{11}{2}}\Rightarrow m=\frac{11}{2}$ $\quad 2m+n=$ ۴

$y=\dfrac{\sqrt{x}+1}{\sqrt{x}-1}\longrightarrow\sqrt{x}=\dfrac{y+1}{y-1}\longrightarrow x=\left(\dfrac{y+1}{y-1}\right)^2\to f^{-1}(x)=\left(\dfrac{x+1}{x-1}\right)^2$

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$(f^{-1})'(x)=2\left(\dfrac{x+1}{x-1}\right)\left(\dfrac{-4}{(x-1)^2}\right)\xrightarrow{x=3}2\times\frac{3}{1}\times\frac{-2}{1}=-12$ ۸

$f(x)\longrightarrow f(0)=1$
$\quad\;\longrightarrow f(-1)=8(-a+1)$ $\Rightarrow\dfrac{1-8(a+1)}{1}=-11\Rightarrow -a+1=\frac{3}{2}\to a=-\frac{1}{2}$

$f(x)=(x^2+1)^3(-\frac{1}{2}x+1)$ ۹

$f'(x)=3(x^2+1)^2(2x)(-\frac{1}{2}x+1)+(x^2+1)^3(-\frac{1}{2})$

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$\xrightarrow[x=1]{x=-2a} f'(1)=3\times4\times2\times\frac{1}{2}+8(-\frac{1}{2})=8$

$x=\frac{\pi}{4}\longrightarrow\frac{\sqrt{2}}{2}\times0=0$
$x=\frac{\pi}{2}\longrightarrow\ln(1)=-1$ $\Bigg\rangle\dfrac{-1-0}{\frac{\pi}{2}-\frac{\pi}{4}}=-\frac{4}{\pi}$

$\begin{cases}y_1(\frac{\pi}{4})=\sin(\frac{\pi}{4})\cos(\frac{\pi}{2})=0\\ y_1(\frac{\pi}{2})=\sin(\frac{\pi}{2})\cos(\pi)=-1\end{cases}\to\dfrac{\Delta y_1}{\Delta x}=\dfrac{-1-0}{\frac{\pi}{2}-\frac{\pi}{4}}=\dfrac{-4}{\pi}$

$y_2=\sin^4x-\cos^4x=-\cos 2x$

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$\begin{cases}y_2(\frac{\pi}{4})=-\cos(\frac{\pi}{2})=0\\ y_2(\frac{\pi}{2})=-\cos(\pi)=1\end{cases}\to\dfrac{\Delta y_2}{\Delta x}=\dfrac{1-0}{\frac{\pi}{2}-\frac{\pi}{4}}=\dfrac{4}{\pi}$ ۱۰

$y=\sin^4x-\cos^4x=(\sin^2x-\cos^2x)\cancel{(\sin^2x+\cos^2x)}\longrightarrow x=\frac{\pi}{4}\to\frac{1}{2}-\frac{1}{2}=0$
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\;\; x=\frac{\pi}{2}=1-0=1$ $\Bigg\rangle\dfrac{1-0}{\frac{\pi}{2}-\frac{\pi}{4}}=\frac{4}{\pi}$

$\longrightarrow\dfrac{\frac{\Delta y_1}{\Delta x}}{\frac{\Delta y_2}{\Delta x}}=\dfrac{-\frac{4}{\pi}}{\frac{4}{\pi}}=-1$