آیلا فیروزنیا ۱۸٫۵

۱. $g(x)=f(\tan^2 x+\sqrt{2}\cos x) \to g'(x)=f'(\tan^2 x+\sqrt{2}\cos x)(1+\tan^4 x-\sqrt{2}\sin x)$

$x=\frac{\pi}{4} \to g'(\frac{\pi}{4})=f'(1+\frac{\sqrt{2}\times\sqrt{2}}{2})(1+1-\frac{\sqrt{2}\times\sqrt{2}}{2}) \to \sqrt{3}=f'(2)\times 3$

$f'(\cancel{2})=\sqrt{3}$ $f'(2)=\frac{\sqrt{3}}{3}$

1.5

۲. $f(x)=\frac{8+\cos^3 x}{4-\cos^2 x}=\frac{(2+\cos x)(4+\cos^2 x-2\cos x)}{(2+\cos x)(2-\cos x)}=\frac{\cos^2 x-2\cos x+4}{2-\cos x}$

$f'(\frac{7\pi}{6})-2g'(\frac{7\pi}{6})=(f-2g)'(\frac{7\pi}{6})$

$f-2g=\frac{\cos^2 x-2\cos x+4-4}{2-\cos x}=\frac{\cos^2 x-2\cos x}{2-\cos x}=\frac{\cos x(\cos x-2)}{2-\cos x}=-\cos x$

$(f-2g)'x=\sin x \to \sin\frac{7\pi}{6}=\frac{-1}{2}$

2

۳. $x>\frac{3}{4}$ $f(x)=(4x-3)\sqrt{ax} \to f'(x)=4\sqrt{ax}+(4x-3)\frac{a}{2\sqrt{ax}}=4\sqrt{ax}$

$x<\frac{3}{4}$ $f(x)=-(4x-3)\sqrt{ax} \to f'(x)=-(4\sqrt{ax}+(4x-3)\frac{a}{2\sqrt{ax}})=-4\sqrt{ax}$

$\to 2(4\sqrt{ax})=2\sqrt{6} \to x=\frac{3}{4} \to 2\sqrt{3a}=\sqrt{6} \to \sqrt{12a}=\sqrt{6}$

$\to 12a=6 \to a=\frac{1}{2}$

2

۴. $y=-ax+2 \to f(x)=y \to f'(x)=y'$

$f(4)=-4a+2$

$\to f(4)+f'(4)=-4a+2-a=-5a+2=-1$

$f'(4)=-a$

$5a=3 \to a=\frac{3}{5}$

2

$f'(4)=-a=\frac{-3}{5}$

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۵-

$7y = x + 5 \to y = \frac{1}{7}x + \frac{5}{7} \to y' = \frac{1}{7} \to \frac{a+3}{(3x+1)^2} = \frac{1}{7}$

$\frac{ax-1}{3x+1} = \frac{1}{7}x + \frac{5}{7} \to \frac{7ax-7}{3x+1} = x + 5 = 3x^2 + 15x + x + 5 = 7ax - 7$

$3x^2 + 16x + 5 = 7ax - 7 \to 3x^2 + (16 - 7a)x + 12 = 0$

$7a + 21 = 9x^2 + 6x + 1 \to 7a = 9x^2 + 6x - 20$

$\to 3x^2 + (16 - 9x^2 - 6x + 20)x + 12 = 0 \to 3x^2 + 16x - 9x^3 - 6x^2 + 20x + 12 = 0$

$9x^3 + 3x^2 - 36x - 12 = 0 \xrightarrow{\div 3} 3x^3 + x^2 - 12x - 4 = 0$

$x^2(3x-1) - 4(3x-1) \to (3x-1)(x^2-4) = 0 \to x = 2$ غ ق ق

$7a = 36 + 12 - 20 \to a = 4$

۶-

$f'(x) = 0 \to$ مماس بر محور x

$\to 3x^2 + a = 0 \to 12 + a = 0 \to a = -12$

$8 - 24 - b = 0 \to b = -16 \to b - a = -16 - (-12) = -4$

۷-

$\lim_{x \to 0} \frac{f(x)f'(x)}{(1-\cos x)^m} = 32\sqrt{2} \to \frac{\sin^n(x^2)\, x^{2n-1}}{(1-\cos x)^m} = 32\sqrt{2}$

$\frac{2^n x^{4n-1}}{x^{2m} \times 2^{-m}} = x^{4n-2m-1} \times 2^m = 32\sqrt{2}$ $\to 4n = 2m+1 \to 4n = 12 \to n = 3$

$2^m = 2^{\frac{11}{2}} \to m = \frac{11}{2}$

$\lim_{x\to 0} 2^{m+1} n x^{4n-2m-1}$ ⟶ حاصل حد عدد متناهی غیرصفر ← توان صفر باشد

$f'(0) = \lim \frac{f(x) - f(0)}{x - 0} = \frac{\sin^n(x^2)}{x} \to \frac{x^{2n}}{x} = x^{2n-1}$

$\begin{cases} 4n - 2m - 1 = 0 \to n = \frac{2m+1}{4} \\ 2^{m+1} n = 32\sqrt{2} \end{cases} \to 2^{m+1} \times \frac{2m+1}{4} = 32\sqrt{2} \to (2m+1)2^{m+1} = 2^7\sqrt{2} \to m = \frac{7}{2}, \; n = 2$ $\quad 2m + n = 9$

$2m + n = 11 + 3 = 14$

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۸. $y = \frac{\sqrt{x}+1}{\sqrt{x}-1} \to \sqrt{x} = \frac{y+1}{y-1} \xrightarrow{\text{توان ۲}} x = \left(\frac{y+1}{y-1}\right)^2$

$f^{-1}(x) = \left(\frac{x+1}{x-1}\right)^2 \to (f^{-1})'(x) = 2\left(\frac{x+1}{x-1}\right)\left(\frac{-2}{(x-1)^2}\right)$

۲

$x=2 \to 2 \times \frac{3}{1} \times \frac{-2}{1} = -12$

۹. $f(x) = (x^2+1)^3(ax+1) \to f(0) = 1$

$f(-1) = 8(-a+1) \to \frac{1-8(-a+1)}{1} = -11$

۲

$8(-a+1) = 12 \to -2a+2 = 3 \to 2a = -1 \to a = \frac{-1}{2}$

$f(x) = (x^2+1)^3\left(\frac{-1}{2}x+1\right)$

$f'(x) = 3(x^2+1)^2(2x)\left(\frac{-1}{2}x+1\right) + \left(\frac{-1}{2}\right)(x^2+1)^3$

$x = -2a \to x = 1 \to 3 \times 4 \times 2 \times \frac{1}{2} + \frac{-1}{2} \times 8 = 12 - 4 = 8$

۱۰. $y = \sin x \cos^2 x \to x = \frac{\pi}{2} \to 1 \times -1 = -1$

$x = \frac{\pi}{4} \to \frac{\sqrt{2}}{2} \times 0 = 0 \to \frac{0-(-1)}{\frac{\pi}{4}} = \frac{4}{\pi}$

۲

$y = \sin^4 x - \cos^4 x \to x = \frac{\pi}{4} \to \frac{1}{4} - \frac{1}{4} = 0$

$x = \frac{\pi}{2} \to 1 - 0 = 1 \to \frac{0-1}{\frac{\pi}{4}} = \frac{-4}{\pi}$

$\frac{\frac{4}{\pi}}{\frac{-4}{\pi}} = \boxed{-1}$