حلیا تابی - ۱۲ دفتر A
۱۴٫۵

(۱)

$$g'(x) = \underbrace{(\tan^2 x+\sqrt{2}\cos x)'}_{①} f'\underbrace{(\tan^2 x+\sqrt{2}\cos x)}_{②}$$

$$① \to 2\tan x(1+\tan^2 x)-\sqrt{2}\sin x \xrightarrow{x=\frac{\pi}{4}} 2(2)-1 = 3$$

$$② \to \tan^2 x+\sqrt{2}\cos x \xrightarrow{x=\frac{\pi}{4}} 1+1=2 .$$

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$$g'(\tfrac{\pi}{4})=\sqrt{3} \qquad 3f'(2)=\sqrt{3} \qquad f'(2)=\frac{\sqrt{3}}{3}$$

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(۲)

$$f'(\tfrac{7\pi}{4})-2g'(\tfrac{7\pi}{4}) \to (f-2g)'(\tfrac{7\pi}{4}) \to f-2g = \frac{8+\cos^3 x}{4-\cos^2 x}-\frac{4}{2-\cos x}$$

$$= \frac{\cancel{(2+\cos x)}(4+\cos^2 x-2\cos x)}{(2-\cos x)\cancel{(2+\cos x)}} - \frac{4}{2-\cos x} \to \frac{\cancel{4}+\cos^2 x-2\cos x-\cancel{4}}{2-\cos x}$$

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$$\to \frac{\cos x(\cancel{\cos x-2})}{\cancel{2-\cos x}} = -\cos x \xrightarrow{\text{مشتق}} +\sin x \xrightarrow{x=\frac{7\pi}{4}} \sin\frac{7\pi}{4} = -\frac{1}{2}$$

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(۳)

$$\begin{cases}(4x-3)\sqrt{ax} & x \geq \frac{3}{4}\\ (-4x+3)\sqrt{ax} & x < \frac{3}{4}\end{cases}$$

$$f'_+(\tfrac{3}{4}) \to 4\sqrt{ax}+\frac{a}{\cancel{2\sqrt{ax}}}\cancel{(4x-3)} = 4\sqrt{\frac{3a}{2}}$$

$$f'_-(\tfrac{3}{4}) \to -4\sqrt{ax}+\frac{a}{\cancel{2\sqrt{ax}}}\cancel{(-4x+3)} = -4\sqrt{\frac{3a}{2}}$$

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$$2\sqrt{3a}-(-2\sqrt{3a}) \Rightarrow \cancel{4}^{2}\sqrt{3a} = \cancel{2}\sqrt{4} \xrightarrow{\text{توان ۲}} 12a=4 \quad a=\frac{1}{3}$$

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(۴)

$$f(4)+f'(4)=-1 \to -4a+2-a=-1 \quad -5a=-3 \quad a=+\frac{3}{5}$$

$$y=-\frac{3}{5}x+2 \xrightarrow[y'=f'(x)]{x=4} f'(4)=-\frac{3}{5}$$

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(۵) ۲ تابع و برخورد می‌کنیم و سپس $\Delta=0$

۱٫۵

$$\frac{ax-1}{3x+1}=\frac{x+5}{7} \qquad 7ax-7=3x^2+x+15x+5$$

$$\xrightarrow[2]{\Delta=0} (16-7a)^2-4(3)(12)=0 \to \begin{cases}a=\frac{4}{7}\\ a=4\end{cases}$$

$$3x^2+x(16-7a)+12=0 \xrightarrow{\Delta=0} 256+49a^2-224a-144=0$$

$$49a^2-224a+112=0 \to 7a^2-32a+16=0 \qquad \frac{32\pm\sqrt{576}}{14}$$

$$\frac{32\pm24}{14} \to \frac{8}{14}, \frac{56}{14} \to (2, 4)$$

$$3x^2+(16-7a)x+12 \xrightarrow{\text{مماس}}$$

$$x=\frac{-b}{2a}=\frac{-16+7a}{6} \to \begin{cases}a=4 \to x=2 & ✓\\ a=\frac{4}{7} \to x=-2 & ✗\end{cases}$$

(۹)

$\longrightarrow f'(۲)=0 \quad ۳x^۲+a=0 \xrightarrow{x=۲} ۱۲+a=0 \quad a=-۱۲$

$f(۲)=0 \longrightarrow ۸+۲\times-۱۲+b=0 \quad b=-۱۶ \quad b-a \to -۱۶+۱۲=-۴$

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(۸)

$f\begin{vmatrix}۲\\۹\end{vmatrix} \longrightarrow f^{-1}\begin{vmatrix}۹\\۲\end{vmatrix} \qquad \frac{\sqrt{x}+1}{\sqrt{x}-1}=۲ \quad ۲\sqrt{x}-۲=\sqrt{x}+1$

$\sqrt{x}=۳ \longrightarrow x=۹$

$\xrightarrow{f'(۹)} \frac{\left(\frac{1}{۲\sqrt{x}}\right)(\sqrt{x}-1)-\left(\frac{1}{۲\sqrt{x}}\right)(\sqrt{x}+1)}{(\sqrt{x}-1)^۲} \xrightarrow{x=۹} -\frac{1}{۱۲} \to$ شیب $f(x)$

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$-۱۲ =$ شیب $f^{-1}(x)$

(۹)

$\longrightarrow \begin{vmatrix}-1\\-۸a+۸\end{vmatrix} \quad \begin{vmatrix}0\\1\end{vmatrix} \qquad m=\frac{1-(-۸a+۸)}{0-(-1)}=\frac{۸a-۷}{1}=-۱۱ \quad ۸a=-۳ \quad a=\frac{-۳}{۸}$

$f'(x) \xrightarrow[x=\frac{-1}{۲}]{x=1} ۳(۲\times1)(1+1)^۲\left(\frac{-1}{۲}\times1+1\right)+\frac{-1}{۲}(1+1)^۳=۸$

۱

$x=-۲a \longrightarrow -۲\times-\frac{۳}{۸}=\frac{۳}{۴} \longrightarrow f'\left(\frac{۳}{۴}\right) \to ۳(x^۲+1)^۲(۲x)(ax+1)+\left(-\frac{۳}{۸}\right)(x^۲+1)^۳$

آهنگ تغییر متوسط $[-1,0]$ $=\frac{f(0)-f(-1)}{0-(-1)}=\frac{1-(۸(-a+1))}{1}=\frac{1-(-۸a+۸)}{1}=۸a-۷=-۱۱ \longrightarrow \begin{cases}a=\frac{-1}{۲}\\x=1\end{cases}$

(۱۰)

$\begin{vmatrix}\frac{\pi}{۴}\\0\end{vmatrix} \quad \begin{vmatrix}\frac{\pi}{۲}\\1\end{vmatrix} \qquad \xrightarrow{\frac{\pi}{۴}} \sin\frac{\pi}{۴}\cos\frac{\pi}{۲}=0 \qquad m=\frac{1}{\frac{1}{\frac{\pi}{۴}}}=\frac{۴}{\pi}$

$\xrightarrow{\frac{\pi}{۲}} \sin\frac{\pi}{۲}\cos\pi=1$

$\sin^۴x-\cos^۴x \to (\sin^۲x-\cos^۲x)(\cancel{\sin^۲x+\cos^۲x}) \to (\sin x-\cos x)(\sin x+\cos x)$

$\frac{\pi}{۴} \to \left(\frac{\sqrt{۲}}{۲}-\frac{\sqrt{۲}}{۲}\right)\left(\frac{\sqrt{۲}}{۲}+\frac{\sqrt{۲}}{۲}\right)=0 \quad \begin{vmatrix}\frac{\pi}{۴}\\0\end{vmatrix} \rightsquigarrow m=\frac{1}{\frac{1}{\frac{\pi}{۴}}}=\frac{۴}{\pi} \xrightarrow{\text{تست}} (۱)$

$\frac{\pi}{۲} \to (1-0)(1+0)=1 \quad \begin{vmatrix}\frac{\pi}{۲}\\1\end{vmatrix}$

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سوال ۷

$$\lim_{x\to 0}\frac{\sin^n(x^2)\times n\sin^{n-1}(x^2)\times \cos(x^2)\times(2x)}{\left(2\sin^2\frac{x}{2}\right)^m}\xrightarrow{\text{هم‌ارزی}}\lim_{x\to 0}\frac{(x^2)^n\, n\,(x^2)^{n-1}\times 2x}{\left(2\left(\frac{x}{2}\right)^2\right)^m}=$$

$$\lim_{x\to 0}\frac{2nx^{2n+2n-2+1}}{\left(\frac{x^2}{2}\right)^m}=\lim_{x\to 0}\frac{2nx^{4n-1}}{\frac{1}{2^m}x^{2m}}=\lim_{x\to 0}2^{m+1}n\,x^{4n-2m-1}$$

حاصل حد عدد غیر متناهی غیر صفر ← توان صفر شود

$$\begin{cases}4n-2m-1=0\longrightarrow n=\frac{2m+1}{4}\\ 2^{m+1}n=32\sqrt{2}\end{cases}\longrightarrow 2^{m+1}\times\frac{2m+1}{4}=32\sqrt{2}\rightarrow(2m+1)2^{m+1}=2^7\sqrt{2}\longrightarrow m=\frac{7}{2}$$

$$n=2$$

$$2m+n=9$$