آدرینا حسینی   تکلیف ش ۲۳   حسابان دوازدهم متن A   ۱۴٫۵

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$$f(x) = |4x-3|\sqrt{ax}$$

$$x=\tfrac{3}{4}^{+} \longrightarrow f'_{+}(x) = \left((4x-3)\sqrt{ax}\right)' = 4\sqrt{ax} + \frac{(4x-3)\times a}{2\sqrt{ax}} = 4\times\sqrt{\frac{a\times 3}{4}}$$

$$x=\tfrac{3}{4}^{-} \longrightarrow f'_{-}(x) = \left((3-4x)\sqrt{ax}\right)' = -4\sqrt{ax} + \frac{(3-4x)\times a}{2\sqrt{ax}} = -4\sqrt{\frac{a\times 3}{4}}$$

اختلاف شیب مماس چپ و راست: $= 8\sqrt{\frac{3a}{4}} = \frac{8\times\sqrt{3a}}{2} = 4\sqrt{3a} = 2\sqrt{6} \longrightarrow 2\sqrt{12a} = 2\sqrt{6} \longrightarrow \boxed{a=\frac{1}{2}}$

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$$y+ax=2 \longrightarrow y=-ax+2 \longrightarrow f'(x)=-a$$

$$f'(2)+f(4)=-1 \longrightarrow -a+-4a+2 = -5a+2=-1 \longrightarrow -5a=-3 \longrightarrow a=\frac{3}{5}$$

$$f'(4) = -a = -\frac{3}{5} = -0.6$$

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$$7y-x=5 \qquad \frac{ax-1}{3x+1} = \longrightarrow \frac{x+5}{7} = \frac{ax-1}{3x+1} \longrightarrow 7ax-7 = 3x^2+16x+5$$

$$\longrightarrow 3x^2+(16-7a)x+12=0 \longrightarrow \Delta=0 \longrightarrow (16-7a)^2-4(3)(12) \longrightarrow (16-7a)^2=(12)^2$$

$$16-7a=12 \longrightarrow a=\frac{4}{7} \quad ✗$$

$$16-7a=-12 \longrightarrow a=4 \quad ✓$$

$$a=4 \longrightarrow 3x^2-12x+12 \longrightarrow 3(x-2)^2=0 \longrightarrow x=2$$

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$$f(x)=x^3+ax-b \longrightarrow f'(x)=3x^2+a \xrightarrow{x=2} 12+a=0 \longrightarrow a=-12$$

$$f(2)=0 \longrightarrow 8-12\times 2-b=0 \longrightarrow b=-16$$

$$b-a=-16+12=-4$$

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$$f(x)=\frac{\sqrt{x}+1}{\sqrt{x}-1} \longrightarrow f'(4) = \frac{\frac{1}{2\sqrt{x}}(\sqrt{x}-1) - (\sqrt{x}+1)\frac{1}{2\sqrt{x}}}{(\sqrt{x}-1)^2} = \frac{\frac{\sqrt{4}-1}{2\sqrt{4}} - \frac{\sqrt{4}+1}{2\sqrt{4}}}{(\sqrt{4}-1)^2}$$

$$\frac{\frac{-2}{2\sqrt{4}}}{(\sqrt{4}-1)^2} = \frac{-1}{\sqrt{4}(\sqrt{4}-1)^2}$$

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آهنگ متوسط $= \frac{\Delta y}{\Delta x} = \frac{(1+1)^3(-a+1)-1}{-1-0} = \frac{-8a+8-1}{-1} = \frac{-8a+7}{-1} = 8a-7 = -11 \longrightarrow 8a=-4 \longrightarrow a=-\frac{1}{2}$

آهنگ لحظه‌ای $x=1$: $= 3(2x)(x^2+1)^2\left(-\frac{1}{2}x+1\right) + (x^2+1)^3\left(-\frac{1}{2}\right) = 6\times 4\times\frac{1}{2} - 4 = 8$

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سوال ۹

$f(x)=\frac{\sqrt{x}+1}{\sqrt{x}-1} \to \sqrt{x}y-y=\sqrt{x}+1 \to \sqrt{x}(y-1)=y+1 \to \sqrt{x}=\frac{y+1}{y-1} \to x=\left(\frac{y+1}{y-1}\right)^2 \to y=\left(\frac{x+1}{x-1}\right)^2$

$f^{-1}(x)=\left(\frac{x+1}{x-1}\right)^2 \to (f^{-1})'=2\left(\frac{x+1}{x-1}\right)\times\frac{-2}{(x-1)^2} \to (f^{-1}(2))'=2\times3\times(-2)=-12$

سوال ۱

$g'(x)=\left(2\tan x(1+\tan^2 x)-\sqrt{2}\sin x\right)f'(\tan^2 x+\sqrt{2}\cos x)$

$\xrightarrow{x=\pi/4} g'(\frac{\pi}{4})=3f'(2)=\sqrt{3} \to f'(2)=\frac{1}{\sqrt{3}} \to f'(2)=\frac{\sqrt{3}}{3}$

سوال ۲

$f'(\frac{7\pi}{6})-2g'(\frac{7\pi}{6})=(f-2g)'(\frac{7\pi}{6})$

$\to (f-2g)'=\left(\frac{8+\cos^3 x}{4-\cos^2 x}-\frac{4}{2-\cos x}\right)'=\left(\frac{(\cos x+2)(\cos^2 x-2\cos x+4)}{(2-\cos x)(2+\cos x)}-\frac{4}{2-\cos x}\right)'=$

$\left(\frac{\cos x(\cos x-2)}{-(\cos x-2)}\right)'=-(\cos x)'=\sin x \xrightarrow{x=7\pi/6} \sin(\frac{7\pi}{6})=\frac{-1}{2}$

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آهنگ متوسط تغییر $=\frac{\Delta y}{\Delta x}=\frac{\sin\frac{\pi}{4}\cos\frac{\pi}{2}-\sin\frac{3\pi}{2}\cos 3\pi}{\frac{\pi}{4}-\frac{\pi}{2}}=\frac{1}{-\frac{\pi}{4}}=\frac{-4}{\pi}$

↳ $\sin x\cos 2x$

آهنگ متوسط تغییر $=\frac{\Delta y}{\Delta x}=\frac{\cancel{\sin^4\frac{\pi}{4}-\cos^4\frac{\pi}{4}}-\sin^4\frac{\pi}{2}+\cancel{\cos^4\frac{\pi}{2}}}{\frac{\pi}{4}-\frac{\pi}{2}}=\frac{-1}{-\frac{\pi}{4}}=\frac{4}{\pi}$

↳ $\sin^4 x-\cos^4 x$

$\left(\frac{\sqrt{2}}{2}\right)^4=\frac{4}{16}=\frac{1}{4}$

$\boxed{\text{۱- برابر است}}$

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سوال ۷

$\lim_{x\to0}\frac{\sin^n(x^2)\times n\sin^{n-1}(x^2)\times\cos(x^2)\times(2x)}{(2\sin^2\frac{x}{2})^m}\xrightarrow{\text{هم‌ارزی}}\lim_{x\to0}\frac{(x^2)^n n(x^2)^{n-1}\times2x}{(2(\frac{x}{2})^2)^m}=$

$\lim_{x\to0}\frac{2nx^{2n+2n-2+1}}{(\frac{x^2}{2})^m}=\lim_{x\to0}\frac{2nx^{4n-1}}{\frac{1}{2^m}x^{2m}}=\lim_{x\to0}2^{m+1}n\,x^{4n-2m-1}$ ⟶ حاصل حد عدد غیر متناهی غیر صفر ← توان صفر

$\begin{cases}4n-2m-1=0 \to n=\frac{2m+1}{4}\\ 2^{m+1}n=32\sqrt{2}\end{cases} \to 2^{m+1}\times\frac{2m+1}{4}=32\sqrt{2} \to (2m+1)2^{m+1}=2^7\sqrt{2} \to m=\frac{7}{2}$

$n=2$

$2m+n=9$