نام و نام خانوادگی ...... فاطمه کریمی ...... ۱۹ پاسخنامه تشریحی تکلیف شماره ۲۳ کلاس دوازدهم A

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$g(x)=f(\tan^2 x+\sqrt{2}\cos x) \Rightarrow g'(x)=f'(\tan^2 x+\sqrt{2}\cos x)(1+\tan^2 x-\sqrt{2}\sin x)$

$x=\frac{\pi}{4} \Rightarrow g'(\frac{\pi}{4})=f'(1+\frac{\sqrt{2}\times\sqrt{2}}{2})(1+1-\frac{\sqrt{2}\times\sqrt{2}}{2})$

$\sqrt{3}=f'(2)\times 1 \Rightarrow \boxed{f'(2)=\sqrt{3}}$

$g'(\frac{\pi}{4})=3f'(2)=\sqrt{3} \rightarrow f'(2)=\frac{1}{\sqrt{3}} \rightarrow f'(2)=\frac{\sqrt{3}}{3}$

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$f(x)=\frac{8+\cos^3 x}{4-\cos^2 x}$ $\qquad g(x)=\frac{2}{2-\cos x}$

$=\frac{(\cos^2 x-2\cos x+4)\cancel{(\cos x+2)}}{(2-\cos x)\cancel{(2+\cos x)}}$

$f'(\frac{7\pi}{6})-2g'(\frac{7\pi}{6})=(f-2g)'(\frac{7\pi}{6})$

$f-2g=\frac{\cos^2 x-2\cos x+\cancel{4-4}}{2-\cos x}=-\cos x \Rightarrow (f-2g)'(x)=\sin x \Rightarrow \sin\frac{7\pi}{6}=-\frac{1}{2}$

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$x>\frac{3}{4} \Rightarrow f(x)=(4x-3)\sqrt{ax} \Rightarrow f'(x)=4\sqrt{ax}+\underbrace{(4x-3)}_{0}\frac{a}{2\sqrt{ax}}=4\sqrt{ax}$

$x<\frac{3}{4} \Rightarrow f(x)=-(4x-3)\sqrt{ax} \Rightarrow f'(x)=-\left(4\sqrt{ax}+\underbrace{(4x-3)}_{0}\frac{a}{2\sqrt{ax}}\right)=-4\sqrt{ax}$

(اختلاف) $2(4\sqrt{ax})=2\sqrt{6} \xrightarrow{x=\frac{3}{4}} 4\sqrt{\frac{3}{4}a}=\sqrt{6} \Rightarrow 16\times\frac{3}{4}a=6 \Rightarrow \boxed{a=\frac{1}{2}}$

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مماس ⇐

$f(x)=y,\ f'(x)=y'$ $\qquad y+ax=2 \Rightarrow y=-ax+2$

$\left.\begin{array}{l} f(4)=-4a+2 \\ f'(4)=-a \end{array}\right\} \Rightarrow f(4)+f'(4)=-5a+2=-1$

$5a=3 \Rightarrow a=\frac{3}{5}$

$f'(4)=-a=\boxed{-\frac{3}{5}}$

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$\left.\begin{array}{l} 7y-x=5 \Rightarrow y=\frac{x}{7}+\frac{5}{7} \Rightarrow y'=\frac{1}{7} \\ y=\frac{ax-1}{3x+1} \Rightarrow y'=\frac{a+3}{(3x+1)^2} \end{array}\right\} \Rightarrow \frac{a+3}{(3x+1)^2}=\frac{1}{7} \Rightarrow a=\frac{(3x+1)^2-21}{7}$

$\frac{x}{7}+\frac{5}{7}=\frac{ax-1}{3x+1} \xrightarrow{\text{جایگذاری}} \frac{x+5}{\cancel{7}}=\frac{x(3x+1)^2-21x-7}{\cancel{7}(3x+1)} \Rightarrow 9x^3+6x^2-20x-7=3x^2+16x+5$

$\Rightarrow 9x^3+3x^2-36x-12=0 \xrightarrow{\div 3} x^2(3x+1)-4(3x+1)=(x^2-4)(3x+1)=0$

قابل قبول $\Rightarrow x=2 \Rightarrow a=\frac{49-21}{7}=\boxed{4}$

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$f(x)=x^3+ax-b \qquad f'(x)=3x^2+a$

(ریشه مضاعف $x=2$ $\Rightarrow f(2)=0 \Rightarrow 8+2a-b=0 \Rightarrow 2a-b=-8$

$f'(2)=0 \Rightarrow 12+a=0 \Rightarrow a=-12 \quad \Big\} \Rightarrow b=-16$

$b-a=-16-(-12)=\boxed{-4}$

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$\lim\limits_{x\to 0}\dfrac{f(x)f'(x)}{(1-\cos x)^m}=32\sqrt{2} \xrightarrow{\text{طبق}*} \dfrac{\sin^n(x^2)\,x^{2n-1}}{(1-\cos x)^m}=32\sqrt{2} \xrightarrow{\text{هم‌ارزی}} \dfrac{x^{4n-1}}{x^{2m}\times 2^{-m}}=x^{4n-2m-1}\times 2^m=32\sqrt{2}$

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$f(x)=\sin^n(x^2)$

$f'(0)=\lim\limits_{x\to 0}\dfrac{f(x)-f(0)}{x-0}=\dfrac{\sin^n(x^2)}{x} \xrightarrow{\text{هم‌ارزی}} \dfrac{x^{2n}}{x}=x^{2n-1}\ *$

x ضریب $=0$

$4n=2m+1 \Rightarrow 4n=12 \Rightarrow n=3$

$2^m=2^{\frac{11}{2}} \Rightarrow m=\frac{11}{2}$

$\Rightarrow 2m+n=11+3=\boxed{14}$

$\begin{cases}4n-2m-1=0 \longrightarrow n=\dfrac{2m+1}{4}\\ 2^{m+1}n=32\sqrt{2}\end{cases} \longrightarrow 2^{m+1}\times\dfrac{2m+1}{4}=32\sqrt{2} \rightarrow (2m+1)2^{m+1}=2^7\sqrt{2} \rightarrow m=\frac{5}{2},\ n=2$

$2m+n=7$

$f(x)=\dfrac{\sqrt{x}+1}{\sqrt{x}-1} \Rightarrow y=\dfrac{\sqrt{x}+1}{\sqrt{x}-1} \Rightarrow \sqrt{x}=\dfrac{y+1}{y-1} \Rightarrow x=\left(\dfrac{y+1}{y-1}\right)^2 \Rightarrow f^{-1}(x)=\left(\dfrac{x+1}{x-1}\right)^2$

$(f^{-1})'(x)=2\left(\dfrac{x+1}{x-1}\right)\left(\dfrac{-2}{(x-1)^2}\right) \xrightarrow{x=2} 2\times\dfrac{3}{1}\times\dfrac{-2}{1}=\boxed{-12}$

۲ ✓ — ۸

$f(x)=(x^2+1)^3(ax+1) \longrightarrow f(0)=1$

$\qquad\qquad \longrightarrow f(-1)=8(-a+1)$

$\Big\} \rightarrow \dfrac{1-8(-a+1)}{1}=-11$

$8(-a+1)=12 \Rightarrow -a+1=\dfrac{3}{2}$

$a=-\dfrac{1}{2}$

$\Rightarrow f(x)=(x^2+1)^3\left(-\tfrac{1}{2}x+1\right)$

$f'(x)=3(x^2+1)^2(2x)\left(-\tfrac{1}{2}x+1\right)+(x^2+1)^3\left(-\tfrac{1}{2}\right)$

$\xrightarrow[x=1]{x=-2a} f'(1)=3\times4\times2\times\tfrac{1}{2}+8\left(-\tfrac{1}{2}\right)=12-4=\boxed{8}$

۲ ✓ — ۹

$y=\sin x\cos 2x \longrightarrow x=\dfrac{\pi}{4} \Rightarrow \dfrac{\sqrt{2}}{2}\times 0=0$

$\qquad \longrightarrow x=\dfrac{\pi}{2} \Rightarrow 1\times(-1)=-1$

$\Big\} \Rightarrow \dfrac{-1-0}{\frac{\pi}{2}-\frac{\pi}{4}}=-4\pi$

$y=\sin^4x-\cos^4x=(\sin^2x-\cos^2x)\underbrace{(\cancel{\sin^2x+\cos^2x})}_{1} \longrightarrow x=\dfrac{\pi}{4} \Rightarrow \dfrac{1}{2}-\dfrac{1}{2}=0$

$\qquad \longrightarrow x=\dfrac{\pi}{2} \Rightarrow 1-0=1$

$\Big\} \Rightarrow \dfrac{1-0}{\frac{\pi}{2}-\frac{\pi}{4}}=4\pi$ — ۱۰

$\dfrac{-4\pi}{4\pi}=\boxed{-1}$ ✓

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