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پاسخ سوالات

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$g'(\frac{\pi}{4}) = f'(\tan^2\frac{\pi}{4} + \sqrt{2}\cos\frac{\pi}{4}) \times (2\tan\frac{\pi}{4} \times (1+\tan^2\frac{\pi}{4}) + -\sqrt{2}\sin\frac{\pi}{4})$

$\Rightarrow \sqrt{3} = f'(2) \times 3 \Rightarrow f'(2) = \frac{\sqrt{3}}{3}$ ✓

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$g'(x) = \frac{-2\sin x}{(2-\cos x)^2}$

$f'(x) = \frac{(-3\sin x\cos^2 x)(4-\cos^2 x) - (1+\cos^3 x)(\sin(2x))}{(4-\cos^2 x)^2}$

$= \cancel{\ldots}$

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$(f-2g)'(x) \Rightarrow \frac{1+\cos^3 x}{4-\cos^2 x} - \frac{4}{2-\cos x} \Rightarrow \frac{1+\cos^3 x - 4(2+\cos x)}{4-\cos^2 x} = \frac{\cos^3 x - 4\cos x}{4-\cos^2 x}$

$= \frac{\cos x(\cos^2 x - 4)}{4-\cos^2 x} \Rightarrow (f-2g)'(x) = \sin x \Rightarrow \sin(\frac{7\pi}{6}) = -\frac{1}{2}$ ✓

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$\lim_{x\to\frac{3}{4}^+} \frac{(4x-3)\sqrt{ax}}{x-\frac{3}{4}} = 2\sqrt{3a}$ $\quad \lim_{x\to\frac{3}{4}^-} \frac{(-(4x-3))(\sqrt{ax})}{x-\frac{3}{4}} = -2\sqrt{3a}$

$\Rightarrow 4\sqrt{3a} = 2\sqrt{12a} = 2\sqrt{6} \Rightarrow a = \frac{1}{2}$ ✓

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$f(4) + f'(4) = -1 \Rightarrow 2 - 4a + (-a) = -1 \Rightarrow 2 - 5a = -1 \Rightarrow 5a = 3 \Rightarrow a = \frac{3}{5}$ ✓

$f'(4) = -a = -\frac{3}{5}$

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$y = \frac{1}{7}x + \frac{a}{7}$

$y' = \frac{a(3x+1) - 3(ax-1)}{(3x+1)^2} = \frac{a+3}{(3x+1)^2} \Rightarrow \frac{a+3}{(3x+1)^2} = \frac{1}{7} \Rightarrow a+3 = \frac{(3x+1)^2}{7}$

$\Rightarrow a = \frac{(3x+1)^2 - 21}{7} = \frac{9x^2+1+6x-21}{7}$

$\frac{x+a}{7} = \frac{ax-1}{3x+1} \Rightarrow 3x^2 + (16-7a)x + 16 = 0$

$\Rightarrow 3x^2 + (16 - 9x^2 - 6x + 20)x + 16 = 0 \Rightarrow -9x^3 - 3x^2 + 36x + 16 = 0 \Rightarrow +3x^3 + x^2 - 12x - 4 = 0$

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$x^2(3x+1) - 4(3x+1) = 0 \Rightarrow (x^2-4)(3x+1) = 0$ $\Rightarrow x = 2 \quad a = \frac{49-21}{7} = 4$ ✓

$x = \pm 2 \quad x = -\frac{1}{3}$

سوال ۷

$$\lim_{x\to 0}\frac{\sin^n(x^2)\times n\sin^{n-1}(x^2)\times\cos(x^2)\times(2x)}{(2\sin^2\frac{x}{2})^m}\xrightarrow{\sin\approx}\lim_{x\to 0}\frac{(x^2)^n n(x^2)^{n-1}\times 2x}{(2(\frac{x}{2})^2)^m}=$$

$$\lim_{x\to 0}\frac{2nx^{2n+2n-2+1}}{(\frac{x^2}{2})^m}=\lim_{x\to 0}\frac{2nx^{4n-1}}{\frac{1}{2^m}x^{2m}}=\lim_{x\to 0}2^{m+1}nx^{4n-2m-1}$$ ⟶ حاصل حد عدد غیرمتناهی غیرصفر ← توان صفر شود

$$\begin{cases}4n-2m-1=0\longrightarrow n=\frac{2m+1}{4}\\ 2^{m+1}n=32\sqrt{2}\end{cases}\longrightarrow 2^{m+1}\times\frac{2m+1}{4}=32\sqrt{2}\to(2m+1)2^{m+1}=2^7\sqrt{2}\longrightarrow m=\frac{7}{2}\qquad 2m+n=9$$

$$n=2$$

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$$f(2)=8+2a-b=0\Rightarrow 2a-b=-8\Rightarrow -24-b=-8$$

$$f'(2)=3(4)+a=0\Rightarrow a=-12\qquad b=-16\qquad\Rightarrow -12+16=-4$$

$$3x^2+a$$

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$$\lim_{x\to 0}\frac{f(x)\,f'(x)}{(1-\cos x)^m}=32\sqrt{2}\Rightarrow\frac{\sin^n(x^n)\times(n\sin^{(n-1)}(x^2))\times\cos(x^2)\times 2x}{(1-\cos x)^m}=32\sqrt{2}$$

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$$f^{-1}(x)\Rightarrow\frac{\sqrt{y}+1}{\sqrt{y}-1}=x\qquad x\sqrt{y}-x=\sqrt{y}+1$$

$$\Rightarrow\sqrt{y}(x-1)=1+x\Rightarrow\sqrt{y}=\frac{1+x}{x-1}\Rightarrow y=\left(\frac{x+1}{x-1}\right)^2$$

$$\Rightarrow y'=2\left(\frac{1+x}{x-1}\right)\times\left(\frac{-1-1}{(x-1)^2}\right)\Rightarrow 2\left(\frac{3}{1}\right)\times\left(\frac{-2}{1}\right)=-12$$

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$$f(0)=1\qquad\Rightarrow\frac{-8a+8-1}{-1}=-11\Rightarrow 11=-8a+7\Rightarrow a=-\frac{1}{2}$$

$$f(-1)=8\times(-a+1)=-8a+8\qquad 4=-8a$$

$$x=-2a=-\frac{1}{2}\times-2=1\Rightarrow f'(x)=3(x^2+1)^2\times 2x\times a(a^{x+1})+a(x^2+1)^3$$

$$\Rightarrow 3(4)\times 2\times\left(\cancel{-\frac{1}{2}+1}\right)+-\cancel{\frac{1}{2}(8)}=8$$

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$$y=\sin^4x-\cos^4x=\sin^2x-\cos^2x$$

$$\Rightarrow\frac{\cancel{\sin\frac{\pi}{4}\cos\frac{\pi}{2}}-\sin\frac{\pi}{2}\cos\pi}{\frac{\pi}{4}-\frac{\pi}{2}}=\frac{-1\times 1\times -1}{\frac{\pi}{4}-\frac{2\pi}{4}}=-\frac{4}{\pi}$$

$$\frac{(\sin^2\frac{\pi}{4}-\cos^2\frac{\pi}{4})-(\sin^2\frac{\pi}{2}-\cos^2\frac{\pi}{2})}{-\frac{\pi}{4}}=\frac{-1(1)}{-\frac{\pi}{4}}=\frac{4}{\pi}\qquad\Rightarrow\frac{-\frac{4}{\pi}}{\frac{4}{\pi}}=-1$$

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