تکلیف ۱۹ سینا شکرنیا

۱- $f(x) = 2\left[\frac{2-x}{2}\right] + a\left[\frac{x+2}{3}\right]$

$$\lim_{x\to(-2)^+} 2\underbrace{\left[\frac{2-x}{2}\right]}_{\substack{2^- \\ 1}} + a\underbrace{\left[\frac{x+2}{3}\right]}_{\substack{0^+ \\ 0}} = 2$$

$x>-2 \to -x<2 \to \frac{-x+2}{2}<2 \qquad x>-2 \to \frac{x+2}{3}>0$

$$\lim_{x\to(-2)^-} 2\underbrace{\left[\frac{2-x}{2}\right]}_{\substack{2^+ \\ 2}} + a\underbrace{\left[\frac{x+2}{3}\right]}_{\substack{0^- \\ -1}} = 4-a$$

$x<-2 \to -x>2 \to \frac{-x+2}{2}>2 \qquad x<-2 \to \frac{x+2}{3}<0$

برای اینکه تابع در نقطه‌ی $x=-2$ حد داشته باشد باید حد راست و چپ آن برابر باشد:

$2 = 4-a \to a=2 \to \left[\frac{a}{3}\right] = \left[\frac{2}{3}\right] =$ صفر

۲- $\lim_{x\to(\frac{\pi}{6})^-} [2\sin x - 1] = [2(\frac{1}{2}^-) - 1] = [0^-] = -1$

۳- $\lim_{x\to -\frac{1}{2}} [8x^3 - x] \xrightarrow{x=-\frac{1}{2}} [8(-\frac{1}{8}) + \frac{1}{2}] = [-\frac{1}{2}] = -1$

۴- $\lim_{x\to(\frac{1}{2})^-} \frac{10x-5+\overbrace{\left[\frac{3}{x^2}\right]}^{11}}{14x - \underbrace{\cancel{\left[\frac{-2}{x^2}\right]}}_{-8}} = \frac{10x-5+11}{14x+8} = \frac{10x+6}{14x+8} = \frac{1}{0^-} = -\infty$

۵- $\lim\limits_{x\to 1^+} \dfrac{\sin^2 \pi x}{[x]+\cos \pi x} = \dfrac{\sin^2 \pi x}{1+\cos \pi x} = \dfrac{0}{0}$

$\dfrac{\sin^2 \pi x}{1+\cos \pi x} \times \dfrac{1-\cos \pi x}{1-\cos \pi x} = \dfrac{\sin^2 \pi x(1-\cos \pi x)}{\underbrace{1-\cos^2 \pi x}_{\sin^2 \pi x}} = 1-\cos \pi x \to 1-(-1) = 2$

۶- $\lim\limits_{x\to -1} \dfrac{\sqrt{2x+3}-\sqrt{3x+4}}{1+\sqrt[3]{x}} \overset{\frac{0}{0}}{\underset{Hop}{=}} \lim\limits_{x\to -1} \dfrac{\frac{2}{2\sqrt{2x+3}}-\frac{3}{2\sqrt{3x+4}}}{\frac{1}{3\sqrt[3]{x^2}}} = \dfrac{1-\frac{3}{2}}{+\frac{1}{3}} = \dfrac{-\frac{1}{2}}{+\frac{1}{3}} = -\dfrac{3}{2}$

۷- $\lim\limits_{x\to 1} \dfrac{2x-7\sqrt{x}+5}{2x-\sqrt{3x+1}} \overset{\frac{0}{0}}{\underset{Hop}{=}} \dfrac{2-\frac{7}{2\sqrt{x}}}{2-\frac{3}{2\sqrt{3x+1}}} = \dfrac{-\frac{3}{2}}{\frac{5}{4}} = \dfrac{-12}{10} = -1.2$

۸- $\lim\limits_{x\to 0} \dfrac{a+\sqrt{bx+c}}{x} = \dfrac{1}{4} \xrightarrow{\text{کسر صفر است}} a+\sqrt{c}=0 \to a=-\sqrt{c}$

$\lim\limits_{x\to 0} \dfrac{\sqrt{bx+c}-\sqrt{c}}{x} \times \dfrac{\text{conj}}{\text{conj}} = \dfrac{\cancel{bx}+\cancel{c}-c}{x(\sqrt{bx+c}+\sqrt{c})} \to \dfrac{b}{2\sqrt{c}} = \dfrac{1}{4}$

$\dfrac{b}{\sqrt{c}} = \dfrac{1}{2} \to b = \dfrac{\sqrt{c}}{2} \qquad \dfrac{ab}{c} = \dfrac{-\sqrt{c}\times\frac{\sqrt{c}}{2}}{c} = \dfrac{-\frac{c}{2}}{c} = -\dfrac{1}{2}$

۹- $\lim\limits_{x\to -2\pi} \dfrac{4+k[\frac{x}{\pi}]}{\sin x} = +\infty \qquad [-k] = -2$

$\lim\limits_{(-2\pi)^+} \dfrac{4+k[\frac{x}{\pi}]}{\sin x} = \dfrac{4-2k}{0^+} = +\infty \to 4-2k>0 \to k<2$

$\dfrac{4}{3}<k<2 \to \boxed{-2<-k<-\dfrac{4}{3}}$

$\lim\limits_{(-2\pi)^-} \dfrac{4+k[\frac{x}{\pi}]}{\sin x} = \dfrac{4-3k}{0^-} = +\infty \to 4-3k<0 \to k>\dfrac{4}{3}$

Scbo

$$\lim_{x\to 8}\frac{b\sqrt{2+\sqrt[3]{x}}-2b}{\underbrace{ax-b}_{8a-b}} = \frac{b\left(\sqrt{2+\sqrt[3]{x}}-2\right)\times مز}{b\left(\frac{1}{8}x-1\right)\times مز} = \frac{2+\sqrt[3]{x}-4}{\left(\frac{1}{8}x-1\right)\left(\sqrt{2+\sqrt[3]{x}}+2\right)} \quad b$$

$$a=\frac{1}{8}b$$

$$=\frac{\sqrt[3]{x}-2\times\left(4+\sqrt[3]{x^2}+2\sqrt[3]{x}\right)}{\frac{1}{8}(x-8)\left(\sqrt{2+\sqrt[3]{x}}+2\right)\left(4+\sqrt[3]{x^2}+2\sqrt[3]{x}\right)} = \frac{x-\cancel{8}}{\frac{1}{8}(x-\cancel{8})\left(\sqrt{2+\sqrt[3]{x}}+2\right)\left(4+\sqrt[3]{x^2}+2\sqrt[3]{x}\right)}$$

$$=\frac{1}{\frac{1}{8}(4)(12)}=\frac{1}{6}$$