تکلیف ۱۹ ۲۰ سید ... نیا

ـ۱

$$f(x) = 2\left[\frac{2-x}{2}\right] + a\left[\frac{x+2}{3}\right]$$

$$\lim_{x\to(-2)^+} 2\underbrace{\left[\frac{2-x}{2}\right]}_{\substack{2^-\\1}} + a\underbrace{\left[\frac{x+2}{3}\right]}_{\substack{0^+\\0}} = 2$$

$$x>-2 \to -x<2 \to \frac{-x+2}{2}<2 \qquad x>-2 \to \frac{x+2}{3}>0$$

$$\lim_{x\to(-2)^-} 2\underbrace{\left[\frac{2-x}{2}\right]}_{\substack{2^+\\2}} + a\underbrace{\left[\frac{x+2}{3}\right]}_{\substack{0^-\\-1}} = 4-a$$

$$x<-2 \to -x>2 \to \frac{-x+2}{2}>2 \qquad x<-2 \to \frac{x+2}{3}<0$$

برای اینکه تابع در نقطه‌ی $x=-2$ حد داشته باشد باید حد راست و چپ آن برابر باشد:

$2 = 4-a \to a=2 \to \left[\frac{a}{3}\right] = \left[\frac{2}{3}\right] =$ صفر

ـ۲ ۲

$$\lim_{x\to(\frac{\pi}{4})^-} [2\sin x - 1] = \left[2\left(\frac{1^-}{2}\right)-1\right] = [0^-] = -1$$

ـ۳ ۲

$$\lim_{x\to-\frac{1}{2}} [8x^3 - x] \xrightarrow{x=-\frac{1}{2}} \left[8\left(-\frac{1}{8}\right)+\frac{1}{2}\right] = \left[-\frac{1}{2}\right] = -1$$

ـ۴ ۲

$$\lim_{x\to(\frac{1}{2})^-} \frac{10x-5+\left[\frac{3}{x^2}\right]^{11}}{14x-\left[-\frac{2}{x^2}\right]_{-8}} = \frac{10x-5+11}{14x+8} = \frac{10x+6}{14x+8} = \frac{1}{0^-} = -\infty$$

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$$\lim_{x\to 1^+}\frac{\sin^2\pi x}{[x]+\cos\pi x}=\frac{\sin^2\pi x}{1+\cos\pi x}=\frac{0}{0}$$

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$$\frac{\sin^2\pi x}{1+\cos\pi x}\times\frac{1-\cos\pi x}{1-\cos\pi x}=\frac{\sin^2\pi x(1-\cos\pi x)}{\cancel{1-\cos^2\pi x}_{\sin^2\pi x}}=1-\cos\pi x$$

$$\downarrow$$

$$1-(-1)=2$$

۶-

$$\lim_{x\to -1}\frac{\sqrt{2x+3}-\sqrt{3x+4}}{1+\sqrt[3]{x}}\ \frac{0}{0}\ \underset{Hop}{=}\ \lim_{x\to -1}\frac{\frac{2}{2\sqrt{2x+3}}-\frac{3}{2\sqrt{3x+4}}}{\frac{1}{3\sqrt[3]{x^2}}}=\frac{1-\frac{3}{2}}{+\frac{1}{3}}=\frac{-\frac{1}{2}}{+\frac{1}{3}}=-\frac{3}{2}$$

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$$\lim_{x\to 1}\frac{2x-7\sqrt{x}+5}{2x-\sqrt{3x+1}}\ \frac{0}{0}\ \underset{Hop}{=}\ \frac{2-\frac{7}{2\sqrt{x}}}{2-\frac{3}{2\sqrt{3x+1}}}=\frac{-\frac{3}{2}}{\frac{5}{4}}=\frac{-12}{10}=-1.2$$

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۸-

$$\lim_{x\to 0}\frac{a+\sqrt{bx+c}}{x}=\frac{1}{4}\ \xrightarrow{\text{کسر } \frac{0}{0} \text{ است}}\ a+\sqrt{c}=0\to a=-\sqrt{c}$$

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$$\lim_{x\to 0}\frac{\sqrt{bx+c}-\sqrt{c}}{x}\times\frac{\text{مزدوج}}{\text{مزدوج}}=\frac{\cancel{bx}+\cancel{c}-\cancel{c}}{\cancel{x}(\sqrt{bx+c}+\sqrt{c})}\to\frac{b}{2\sqrt{c}}=\frac{1}{4}$$

$$\frac{b}{\sqrt{c}}=\frac{1}{2}\to b=\frac{\sqrt{c}}{2}\qquad \frac{ab}{c}=\frac{-\sqrt{c}\times\frac{\sqrt{c}}{2}}{c}=\frac{-\frac{c}{2}}{c}=\frac{1}{2}$$

۹-

$$\lim_{x\to -2\pi}\frac{4+k\left[\frac{x}{\pi}\right]}{\sin x}=+\infty\qquad [-k]=-2$$

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$$\lim_{(-2\pi)^+}\frac{4+k\left[\frac{x}{\pi}\right]}{\sin x}=\frac{4-2k}{0^+}=+\infty\to 4-2k>0\to k<2$$

$$\frac{4}{3}<k<2$$

$$\boxed{-2<-k<-\frac{4}{3}}$$

$$\lim_{(-2\pi)^-}\frac{4+k\left[\frac{x}{\pi}\right]}{\sin x}=\frac{4-3k}{0^-}=+\infty\to 4-3k<0\to k>\frac{4}{3}$$

$$\lim_{x\to 8}\frac{b\sqrt{2+\sqrt[3]{x}}-2b}{ax-b}=\frac{b\left(\sqrt{2+\sqrt[3]{x}}-2\right)\times\text{مز}}{b\left(\frac{1}{8}x-1\right)\times\text{مز}}=\frac{2+\sqrt[3]{x}-4}{\left(\frac{1}{8}x-1\right)\left(\sqrt{2+\sqrt[3]{x}}+2\right)}\quad b$$

$$\underbrace{ax-b}_{8a-b}$$

$$a=\frac{1}{8}b$$

$$=\frac{\sqrt[3]{x}-2\times\left(4+\sqrt[3]{x^2}+2\sqrt[3]{x}\right)}{\frac{1}{8}(x-8)\left(\sqrt{2+\sqrt[3]{x}}+2\right)\left(4+\sqrt[3]{x^2}+2\sqrt[3]{x}\right)}=\frac{x-\cancel{8}}{\frac{1}{8}(x-\cancel{8})\left(\sqrt{2+\sqrt[3]{x}}+2\right)\left(4+\sqrt[3]{x^2}+2\sqrt[3]{x}\right)}$$

$$=\frac{1}{\frac{1}{8}(4)(12)}=\frac{1}{6}$$ ✓

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