(1)

$$\lim_{x\to -2^+} f(x) = 2\left[\frac{2-(-2^+)}{2}\right] + a\left[\frac{-2^+ + 2}{3}\right] = \overbrace{2\times\left[2^-\right]}^{2\times 1} + \overbrace{a\left[0^+\right]}^{a\times 0} = 2$$

$$\lim_{x\to -2^-} f(x) = 2\left[\frac{2-(-2^-)}{2}\right] + a\left[\frac{-2^- + 2}{3}\right] = 2\times\left[2^+\right] + a\left[0^-\right] = 4-a$$

$$\Rightarrow 2 = 4-a \Rightarrow a=2 \Rightarrow \left[\frac{a}{3}\right] = \left[\frac{2}{3}\right] = \boxed{0}$$

(2)

$$\lim_{x\to (\frac{\pi}{6})^-} \left[2\sin x - 1\right] = \left[2\times\left(\tfrac{1}{2}\right)^- - 1\right] = \left[0^-\right] = \boxed{-1}$$

(3)

$$\lim_{x\to -\frac{1}{2}} \left[8x^3 - x\right] = \left[8\times\left(-\tfrac{1}{8}\right) + \tfrac{1}{2}\right] = \left[-\tfrac{1}{2}\right] = \boxed{-1}$$

(4)

$$\lim_{x\to (-\frac{1}{2})^-} \frac{10x - 5 + \left[\frac{3}{x^2}\right]}{14x - \left[-\frac{2}{x^2}\right]} = \frac{10x-5-11}{14x+8} = \frac{1}{0^-} = \boxed{-\infty}$$

$$x < -\frac{1}{2} \Rightarrow x^2 > \frac{1}{4} \Rightarrow \frac{1}{x^2} < 4 \Rightarrow \frac{-2}{x^2} > -8 \quad \text{و} \quad \frac{3}{x^2} < 12$$

(5)

$$\lim_{x\to 1^+} \frac{\sin^2 \pi x}{[x] + \cos \pi x} = \frac{\sin^2 \pi x}{1+\cos \pi x} = \frac{1-\cos^2 \pi x}{1+\cos \pi x} = \frac{(1+\cos\pi x)(1-\cos\pi x)}{(1+\cos\pi x)}$$

$$= \frac{1-\cos\pi x}{1} \Rightarrow \lim_{x\to 1^+} 1-\cos\pi x = 1+1 = \boxed{2}$$

(6)

$$\lim_{x\to -1} \frac{\sqrt{2x+3} - \sqrt{3x+4}}{1+\sqrt[3]{x}} = \frac{2x+3-3x-4}{1+x} \times \frac{3}{2} = \frac{-(1+x)}{1+x} \times \frac{3}{2}$$

$$= \boxed{-\frac{3}{2}}$$

رونیا الوتی

⑦ $$\lim_{x\to 1}\frac{2x-7\sqrt{x}+5}{2x-\sqrt{3x+1}}=\frac{(2x+5)^2-49x}{4x^2-3x-1}=\frac{4x^2+20x-49x+25}{4x^2-3x-1}$$

$$=\frac{(x-1)(x-\frac{25}{4})}{(x-1)(x+\frac{1}{4})}=\boxed{-\frac{21}{5}}$$

⑧ $$\lim_{x\to 0}\frac{a+\sqrt{bx+c}}{x}=\frac{1}{4}\Rightarrow\frac{\frac{b}{2\sqrt{bx+c}}}{1}=\frac{b}{2\sqrt{bx+c}}=\frac{1}{4}$$

$$\Rightarrow a+\sqrt{c}=0\Rightarrow a=-\sqrt{c} \quad ①$$

$$\xrightarrow{x=0}\frac{b}{2\sqrt{c}}=\frac{1}{4}\Rightarrow 2b=\sqrt{c}\xrightarrow{\text{طبق } ①}2b=\sqrt{c}=-a$$

$$\frac{ab}{c}=\frac{-2b\times b}{4b^2}=\boxed{-\frac{1}{2}}$$

⑨ $$\lim_{x\to -2\pi}\frac{4+k[\frac{x}{\pi}]}{\sin x}=+\infty\xrightarrow{x\to -2\pi^+}\frac{4+k(-2)}{0^-}=+\infty$$

$$\Rightarrow 4-2k<0\Rightarrow k>2 \quad ①$$

$$\xrightarrow{x\to -2\pi^-}\frac{4+k(-3)}{0^+}=+\infty\Rightarrow 4-3k>0\Rightarrow k<\frac{4}{3} \quad ②$$

$$\xrightarrow[①,②]{\text{طبق}}\quad \frac{4}{3}<k<2\Rightarrow\boxed{[-k]=-2}$$

⑩ به ازای $x=8$ صورت حد صفر می‌شود.

$$8a-b=0\Rightarrow 8a=b$$

$$\xrightarrow[\text{هوپیتال}]{HOP}\frac{\frac{\frac{b}{3\sqrt[3]{x^2}}}{2\sqrt{2+\sqrt[3]{x}}}}{a}\xrightarrow{x=8}\frac{b}{48a}=\frac{8a}{48a}=\boxed{\frac{1}{6}}$$