دوازدهم دفتر A

پرنیان شمشیری _ تکلیف شماره ۱۹

سوال ۱)

$$x=-2 \to -2^{+}: \lim_{x\to -2^{+}} 2\left[\frac{2-x}{2}\right]+a\left[\frac{x+2}{3}\right] = 2[2^{-}]+a[0^{+}] = 2(1)+0=2$$

$$x=-2 \to -2^{-}: \lim_{x\to -2^{-}} 2\left[\frac{2-x}{2}\right]+a\left[\frac{x+2}{3}\right] = 2[2^{+}]+a[0^{-}] = 2(2)+(-a) = 4-a$$

$$4-a=2 \to a=2$$

$$\to \left[\frac{a}{3}\right] = \left[\frac{2}{3}\right] = 0$$

سوال ۲)

$$\lim_{x\to(\frac{\pi}{6})^{-}} [2\sin x-1] \to x<\frac{\pi}{6} \to \sin x<\frac{1}{2} \to 2\sin x<1 \to 2\sin x-1<0$$

$$\to \lim_{x\to(\frac{\pi}{6})^{-}} [2\sin x-1] = \lim[0^{-}] = -1$$

سوال ۳)

$$\lim_{x\to -\frac{1}{2}} [8x^3-x] = \lim\left[8\left(-\frac{1}{8}\right)-\left(-\frac{1}{2}\right)\right] = \lim\left[-1+\frac{1}{2}\right] = \lim\left[-\frac{1}{2}\right] = -1$$

سوال ۴)

$$\lim_{x\to(-\frac{1}{2})^{-}} \frac{10x-5+\left[\frac{3}{x^2}\right]}{14x-\left[\frac{-2}{x^2}\right]} = \frac{-5-5+12}{-8-7} = \frac{2}{-15}$$

$$x<-\frac{1}{2} \to x^2>\frac{1}{4} \to \frac{3}{x^2}>\frac{3}{\frac{1}{4}} \to \frac{3}{x^2}>12 \to \left[\frac{3}{x^2}\right]=12$$

$$x<-\frac{1}{2} \to x^2>\frac{1}{4} \to \frac{2}{x^2}>\frac{2}{\frac{1}{4}} \to \frac{2}{x^2}>8 \to \frac{-2}{x^2}<8 \to \left[\frac{-2}{x^2}\right]=7$$

سوال ۵)

$$\lim_{x\to 1^{+}} \frac{\sin^2\pi x}{[x]+\cos\pi x} = \frac{0}{1+(-1)} = \frac{0}{0} \underset{\text{رفع ابهام}}{=} \frac{1-\cos^2\pi x}{1+\cos\pi x} = \frac{(1+\cos\pi x)(1-\cos\pi x)}{1+\cos\pi x} = 1-(-1) = 2$$

$$x>1 \to \pi x>1 \to \sin^2\pi x<1 \to \sin^2\pi x\simeq 0$$

$$x>1 \to \pi x>1 \to \cos\pi x \neq 1 \to \cos\pi \simeq -1$$

سوال ۶)

$$\lim_{x\to -1}\frac{\sqrt{2x+3}-\sqrt{3x+4}}{1+\sqrt[3]{x}}=\frac{0}{0}\xrightarrow{\text{رفع ابهام}}\times\frac{\sqrt{2x+3}+\sqrt{3x+4}}{\sqrt{2x+3}+\sqrt{3x+4}}\times\frac{1+\sqrt[3]{x^2}-\sqrt[3]{x}}{1+\sqrt[3]{x^2}-\sqrt[3]{x}}=\frac{\overbrace{\big((2x+3)-(3x+4)\big)}^{\cancel{2x+3-3x-4}=-x-1=-\cancel{(x+1)}}\big(1+\sqrt[3]{x^2}-\sqrt[3]{x}\big)}{\big(\sqrt{2x+3}+\sqrt{3x+4}\big)\cancel{(1+x)}}$$

$$=\frac{(-1)\big(1+1-(-1)\big)}{(1+1)}=\frac{(3)(-1)}{2}=-\frac{3}{2}$$

سوال ۷)

$$\lim_{x\to 1}\frac{2x-7\sqrt{x}+5}{2x-\sqrt{3x+1}}=\frac{0}{0}\xrightarrow{\text{رفع ابهام}}\times\frac{2x+\sqrt{3x+1}}{2x+\sqrt{3x+1}}\times\frac{2x+5+7\sqrt{x}}{2x+5+7\sqrt{x}}=\frac{(4x^2+25+20x-49x)(2x+\sqrt{3x+1})}{(4x^2-3x-1)(2x+5+7\sqrt{x})}$$

$$=\frac{(4x^2-29x+25)(2x+\sqrt{3x+1})}{(4x^2-3x-1)(2x+5+7\sqrt{x})}=\frac{\cancel{(x-1)}(4x-25)(2x+\sqrt{3x+1})}{\cancel{(x-1)}(4x+1)(2x+5+7\sqrt{9})}=\frac{(-21)(2+2)}{(-3)(14)}=\frac{\cancel{-21}^{\,7\;1}\times 4}{\cancel{-3}_{1}\times\cancel{14}_{2}}=2$$

سوال ۸)

$$\lim_{x\to 0}\frac{a+\sqrt{bx+c}}{x}=\frac{0}{0}\to a+\sqrt{c}=0\to a=-\sqrt{c}\to a^2=c$$

$$\xrightarrow{\text{رفع ابهام}}\frac{a^2-bx-\cancel{c}}{(x)(a-\sqrt{bx+c})}\xrightarrow{a^2=c}\frac{-b\cancel{x}}{\cancel{(x)}(a-\sqrt{bx+c})}=\frac{-b}{a-\sqrt{bx+c}}\xrightarrow{x=0}\frac{-b}{a-\sqrt{c}}=\frac{1}{4}\xrightarrow{a=-\sqrt{c}}\frac{-b}{2a}=\frac{1}{4}$$

$$\longrightarrow b=\frac{-1}{2}a\ ,\ c=a^2\to\frac{ab}{c}=\frac{a\times\frac{-1}{2}a}{a^2}=\frac{-1}{2}$$