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نرگس زینعلی

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$x \to -2^+ \longrightarrow f(x) = 2\left[\frac{2-(-2^+)}{2}\right] + a\left[\frac{(-2^+)+2}{3}\right] = 2 + a(0) = 2$

$x \to -2^- \longrightarrow f(x) = 2\left[\frac{2-(-2^-)}{2}\right] + a\left[\frac{(-2^-)+2}{3}\right] = 4 + a(-1) = 2$

$4 - a = 2 \Rightarrow a = 2$ $\quad \Rightarrow \left[\frac{0^+}{3}\right] = \left[\frac{2}{\ldots}\right] = 0$

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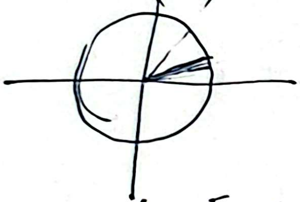
$\lim\limits_{x \to \left(\frac{5\pi}{6}\right)^-} [2\sin x - 1] = \lim\limits_{x \to \left(\frac{5\pi}{6}\right)^-} \left[2 \times \left(\frac{1}{2}^-\right) - 1\right] = \lim\limits_{x \to \left(\frac{5\pi}{6}\right)^-} [1^- - 1] = -1$



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$\lim\limits_{x \to -\frac{1}{2}} [8x^3 - x] = \lim\limits_{x \to -\frac{1}{2}} [x(8x^2 - 1)]$

$\longrightarrow x \to -\frac{1}{2}^+ \longrightarrow \lim\limits_{x \to -\frac{1}{2}^+} \left[-\frac{1}{2}^+\left(\frac{1}{4} \times 8 - 1\right)\right] = \lim\limits_{x \to -\frac{1}{2}^+} \left[-\frac{1}{2}^+ (1^+)\right] = \lim\limits_{x \to -\frac{1}{2}^+} \left[\left(-\frac{1}{2}\right)^+\right] = -1$

$\longrightarrow x \to -\frac{1}{2}^- \longrightarrow \lim\limits_{x \to -\frac{1}{2}^-} \left[-\frac{1}{2}^-\left(\frac{1}{4} \times 8 - 1\right)\right] = \lim\limits_{x \to -\frac{1}{2}^-} \left[\left(-\frac{1}{2}\right)^-\right]$

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سوال ۴)

$$x \to \left(-\tfrac{1}{2}\right)^- \longrightarrow x < -\tfrac{1}{2} \to x^2 > \tfrac{1}{4} \to \frac{1}{x^2} < 4 \to \frac{3}{x^2} < 12 \to \left[\frac{3}{x^2}\right] = 11$$

$$\frac{1}{x^2} < 4 \to -\frac{1}{x^2} > -4 \to \frac{-2}{x^2} > -8 \to \left[\frac{-2}{x^2}\right] = -8$$

$$\rightsquigarrow \lim_{x \to (-\frac{1}{2})^-} = \frac{10x - 5 + 11}{16x - (-8)} = \lim_{x \to (-\frac{1}{2})^-} = \frac{10x + 6}{16x + 8} = \frac{-5 + 6}{(-8)^- + 8} = \boxed{\frac{1}{0^-}} = \boxed{-\infty}$$

$$\lim_{x \to 1^+} \frac{\sin^2(\pi x)}{\cos(\pi x) + [x]} = \frac{\sin^2(\pi^+)}{\cos(\pi^+) + 1} \overset{\text{هم‌ارزی}}{\approx} \frac{\pi^2}{1 - \frac{\pi^2}{2} + 1} = \frac{\pi^2}{\frac{\pi^2}{2}} = 2$$

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$$\lim_{x \to -1} \frac{\sqrt{2x-3} - \sqrt{3x+4}}{1+\sqrt[3]{x}} \times \frac{\sqrt{2x+3} + \sqrt{3x+4}}{\sqrt{2x+3} + \sqrt{3x+4}} \times \frac{\left(1-\sqrt[3]{x}+\sqrt[3]{x^2}\right)}{\left(1+\sqrt[3]{x^2}-\sqrt[3]{x}\right)}$$

$$= \frac{2x-3-3x-4}{1+x} \times \frac{1-\sqrt[3]{x}+\sqrt[3]{x^2}}{\left(\sqrt{2x+3}+\sqrt{3x+4}\right)} = -1 \times \frac{\left(1-(-1)+1\right)}{\sqrt{1}+\sqrt{1}} = -1 \times \frac{3}{2} = -\frac{3}{2}$$

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$$\lim_{x \to 1} \frac{2x - 7\sqrt{x} + 5}{2x - \sqrt{3x+1}} = \frac{2 - \frac{7}{2\sqrt{x}}}{2 - \frac{3}{2\sqrt{3x+1}}} = \frac{2 - \frac{7}{2}}{2 - \frac{3}{2\sqrt{4}}} = \frac{-\frac{3}{2}}{\frac{5}{4}} = -\frac{6}{5}$$

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$$a+\sqrt{bx+c}=0 \;\xrightarrow{x=0}\; a+\sqrt{c}=0 \Rightarrow a=-\sqrt{c} \Rightarrow a^2=c$$

$$\Rightarrow \frac{a+\sqrt{bx+c}}{x}\times\frac{a-\sqrt{bx+c}}{a-\sqrt{bx+c}} = \frac{a^2-\left(\sqrt{bx+c}\right)^2}{x\left(a-\sqrt{bx+c}\right)}=\frac{1}{4} \Rightarrow \frac{(bx-c)\cancel{-a^2}\to c}{x\left(\sqrt{bx+c}-a\right)}=\frac{1}{4}$$

$$\Rightarrow \frac{b\cancel{x}}{\cancel{x}\left(\sqrt{bx+c}-a\right)}=\frac{1}{4}$$

$$\Rightarrow \frac{b}{\sqrt{bx+c}-a}\;\xrightarrow{x=0}\;=\frac{1}{4} \Rightarrow 4b=\sqrt{c}-a \Rightarrow 4b=2\sqrt{c} \Rightarrow 4b=-2a \Rightarrow b=-\frac{1}{2}a$$

$$\Rightarrow \frac{ab}{c}=\frac{-\frac{1}{2}a\times a}{a^2}=-\frac{1}{2}\ \checkmark$$

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$$(I)\cap(II) \to \frac{4}{3} < k < 2 \to -2 < -k < -\frac{4}{3} \to [-k] = \boxed{-2}$$

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$$x \to -2\pi^{+} \to \left[\frac{-2\pi^{+}}{\pi}\right] = -2 \longrightarrow \lim \frac{4-2k}{0^{-}} = +\infty \Rightarrow 4 < 2k \Rightarrow k > 2 \quad (I)$$

$$\Rightarrow \sin x = 0^{-}$$

$$x \to -2\pi^{-} \to \left[\frac{-2\pi^{-}}{\pi}\right] = -3 \to \lim \frac{4-3k}{0^{-}} = +\infty \Rightarrow 4 < 3k \Rightarrow k > \frac{4}{3} \quad (II)$$

$$\Rightarrow [-k] = -3$$

$$\Rightarrow k > 2$$

$$\lim_{x\to 8} \frac{b\sqrt{2+\sqrt[3]{x}} - 2b}{ax-b} = \frac{b\sqrt{\cancel{2+2}}^{\,4} - 2b}{8a-b} = \frac{2b-2b}{8a-2b} \to 8a = 2b$$

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سوال ۱۰

$$8a - b = 0 \to b = 8a$$

$$\lim_{x\to 8} \frac{8a\left(\sqrt{2+\sqrt[3]{x}} - 2\right)}{ax - 8a} = \frac{8a\left(\sqrt{2+\sqrt[3]{x}} - 2\right)}{ax-8a} \times \frac{\sqrt{2+\sqrt[3]{x}}+2}{\sqrt{2+\sqrt[3]{x}}+2} =$$

$$\lim_{x\to 8} \frac{8a\left(\sqrt[3]{x} - 2\right)}{a(x-8)\times 4} \to \lim_{x\to 8} \frac{2\left(\sqrt[3]{x}-2\right)}{\left(\sqrt[3]{x}-2\right)\left(\sqrt[3]{x^2}+4+2\sqrt[3]{x}\right)} = \frac{2}{12} = \boxed{\frac{1}{6}}$$