«مبینا خالقی الحسینی» «دوازدهم دختر A. [illegible]»

۱ـ

الف) $\lim\limits_{x\to 2^+} \dfrac{2x+5}{x+1} = \dfrac{9}{3} = 3$

ب) $\lim\limits_{x\to 2^-} \dfrac{2x+5}{x+1} = \dfrac{9}{3} = 3$

ج) $\lim\limits_{x\to 2} \dfrac{2x+5}{x+1} = \dfrac{9}{3} = 3$

د) $\lim\limits_{x\to 0} \dfrac{2x+7}{[x^2]}$ $\begin{cases} 0^+ : \dfrac{7}{[0^+]} = \text{ت ن} \\ 0^- : \dfrac{7}{[0^+]} = \text{ت ن} \end{cases}$

پس این عبارت در $x\to 0$ حد ندارد

۲ـ

الف) $\lim\limits_{x\to 3^-} 4[x]-2 = 4\overbrace{[3^-]}^{2}-2 = 6$

ب) $\lim\limits_{x\to 3^-} [4x-2] = [10^-] = 9$

ج) $\left[\lim\limits_{x\to 3^-} 4x-2\right] = [10] = 10$

د) $\left[\lim\limits_{x\to 3^+} 4x-2\right] = [10] = 10$

۳ـ

الف) $\lim\limits_{x\to 3^-} \dfrac{3\overbrace{[x]}^{[3^-]=2}+1}{2} = \dfrac{7}{2}$

ب) $\lim\limits_{x\to 3^-} \left[\dfrac{3x+1}{2}\right] = [5^-] = 4$

ج) $\left[\lim\limits_{x\to 3} \dfrac{3x+1}{2}\right] = 5$

د) $\left[\lim\limits_{x\to 3^+} \dfrac{3x+1}{2}\right] = 5$

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الف) $\lim\limits_{x\to 1} \dfrac{x-5}{x^2-6x+5} = \dfrac{-4}{0}$

$1^+ : \dfrac{-4}{0^-} = +\infty$

$1^- : \dfrac{-4}{0^+} = -\infty$ ⇒ حد ندارد

مشتق $y' = 2x-6 \xrightarrow{x=1} -4<0$

تعیین علامت تابع $x^2-6x+5=(x-1)(x-5)$

$$\begin{array}{ccccc} & 1 & & 5 & \\ + & | & - & | & + \end{array}$$

ب) $\lim\limits_{x\to 3} \dfrac{x-4}{x^2-6x+9} = \dfrac{-1}{0}$

$3^+ : \dfrac{-1}{0^+} = -\infty$ ⇒ حد ندارد

$3^- : \dfrac{-1}{0^+} = -\infty$

ریشه مضاعف نزن!

$(x-3)^2$

$$\begin{array}{ccc} & 3 & \\ + & | & + \end{array}$$

۵_

الف) $\lim\limits_{x\to -3} [-3x] - [5x]$

$-3^+ : [9^-] - [-15^+] = 8-(-15) = \boxed{23}$

$-3^- : [9^+] - [-15^-] = 9-(-16) = \boxed{25}$

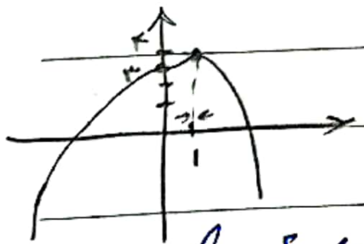
ب) $\lim\limits_{x\to (\frac{-1}{2})^-} \left[\dfrac{-1}{x^4}\right] = \left[\dfrac{-1}{(\frac{1}{16})^+}\right] = \left[\dfrac{\ominus 1}{\frac{1}{16^+}}\right] = [\ominus 16^-] = -17$

۶_

الف) $\lim\limits_{x\to 1} [4x^3-3x^4+3] = [4^-] = 3$

مشتق: $12x^2-12x^3 = 12x^2(1-x)$

$$\begin{array}{ccccc} & 0 & & 1 & \\ + & | & + & | & - \end{array}$$



ب) $\lim\limits_{x\to 0} [4x^3-3x^4+3]$

$0^+ : [3^+] = 3$ ⇒ حد ندارد

$0^- : [3^-] = 2$

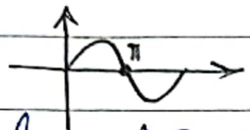
$$\frac{\ \ \ 2\ \ \ }{-\ \ \ \ +}$$

۷ـ الف) $\lim_{x\to 2} \frac{|x^3-8|}{x^2-4}$

$2^+ : \lim_{x\to 2^+} \frac{\overbrace{x^3-8}^{(\cancel{x-2})(x^2+2x+4)}}{\underbrace{x^2-4}_{(\cancel{x-2})(x+2)}} = \frac{12}{4} = \boxed{3}$

$2^- : \lim_{x\to 2^-} \frac{\overbrace{-(x^3-8)}^{-(\cancel{x-2})(x^2+2x+4)}}{\underbrace{x^2-4}_{(\cancel{x-2})(x+2)}} = \frac{-12}{4} = \boxed{-3}$

$\Rightarrow$ حد ندارد



ب) $\lim_{x\to \pi} \frac{|\sin x|}{\sin 2x}$

$\pi^+ : \lim_{x\to\pi^+} \frac{-\cancel{\sin x}}{2\cancel{\sin x}\cos x} = \frac{-1}{-1} = \boxed{1}$

$\pi^- : \lim_{x\to\pi^-} \frac{\cancel{\sin x}}{2\cancel{\sin x}\cos x} = \frac{1}{-1} = \boxed{-1}$

$\Rightarrow$ حد ندارد

۸ـ الف) $\lim_{x\to \frac{3\pi}{2}} \sqrt{\cos x}$

$\frac{3\pi}{2}^+ : \sqrt{0^+} = 0$

$\frac{3\pi}{2}^- : \sqrt{0^-} =$ ت ن

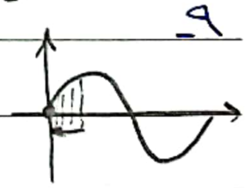
ب) $\lim_{x\to 0} \sqrt{x-2\sqrt{x}}$

$0^+ : \sqrt{x-2\sqrt{0^+}}$ ت ن

مثلاً $x=\frac{1}{100} \to \sqrt{\frac{1}{100}-\frac{2}{10}} = \sqrt{\frac{1}{100}-\frac{20}{100}}$ ⊝

$0^- : \sqrt{x-2\sqrt{0^-}} =$ ت ن

۹ـ الف) $\lim_{x\to 0^+} \frac{(-1)^{[\sin x]}}{x^3-x^2} = \lim_{x\to 0^+} \frac{(-1)^{[\sin 0^+]}}{x^3-x^2}$, $[\sin 0^+] \to [0^+] = 0$



$\lim_{x\to 0^+} \frac{1}{x^3-x^2} = \frac{1}{0^-} = -\infty$

$y' = 3x^2-2x < 0$ ن

ب) $\lim\limits_{x\to 2} \dfrac{x^2-[x^2]}{x-[x]}$

$2^+:\ \lim\limits_{x\to 2^+} \dfrac{x^2-[4^+]}{x-[2^+]} =$

$\lim\limits_{x\to 2^+} \dfrac{x^2-4}{x-2} = \lim\limits_{x\to 2^+} \dfrac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}} = 4$

$2^-:\ \lim\limits_{x\to 2^-} \dfrac{x^2-[4^-]}{x-[2^-]} =$ $\Rightarrow$ حد ندارد

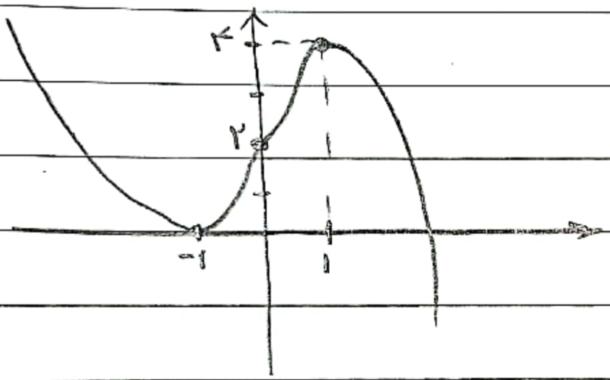
$\lim\limits_{x\to 2^-} \dfrac{x^2-3}{x-1} = \dfrac{1}{1} = 1$

۱۰- الف) $y = 5x^3 - 3x^5 + 2$

$y' = -15x^4 + 15x^2 = -15x^2(x^2-1)$

$\downarrow\ 0 \qquad \downarrow\ \pm 1$

	-1		0 *		$+1$	
$-$		$+$		$+$		$-$
$\searrow$	0	$\nearrow$	2	$\nearrow$	4	$\searrow$

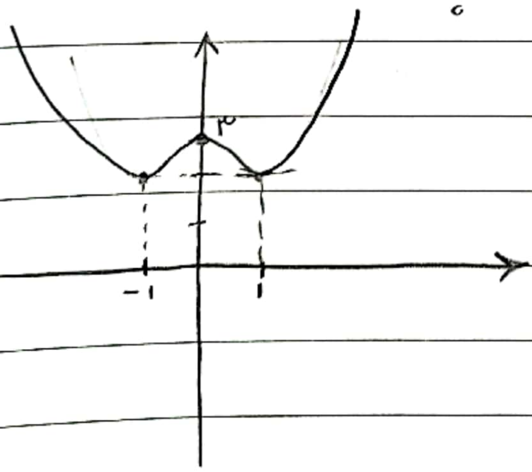


ب) $y = x^4 - 2x^2 + 3$

$y' = 4x^3 - 4x = 4x(x^2-1)$

$\downarrow\ 0 \qquad \downarrow\ \pm 1$

	-1		0		$+1$	
$-$		$+$		$-$		$+$
$\searrow$	2	$\nearrow$	3	$\searrow$	2	$\nearrow$



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