

کلاس دوازدهم چه وقت است؟ الف

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1

1.5

الف) $\lim_{x \rightarrow 2^+} \frac{2x+2}{x+1} = \frac{4}{3} = \mu$ ✓

ب) $\lim_{x \rightarrow 2^-} \frac{2x+2}{x+1} = \frac{4}{3} = \mu$ ✓

ج) $\lim_{x \rightarrow 2} \frac{2x+2}{x+1} = \frac{4}{3} = \mu$ $\left\{ \begin{array}{l} \lim_{x \rightarrow 2^+} \frac{2x+2}{x+1} = \mu \\ \lim_{x \rightarrow 2^-} \frac{2x+2}{x+1} = \mu \end{array} \right.$ ✓

د) $\lim_{x \rightarrow 0} \frac{2x+V}{[x]}$ $\left\{ \begin{array}{l} \rightarrow 0^+ \rightarrow 0 \\ \rightarrow 0^- \rightarrow \frac{2x+V}{-1} = -V \end{array} \right.$

$\lim_{x \rightarrow 0} \frac{2x+V}{[x]} = \lim_{x \rightarrow 0} \frac{2(0)+V}{[0]}$ تعریف نشده

خرج صفر دیت می شود

الف) $\lim_{x \rightarrow 2^-} f[x] - 2 = f(2) - 2 = 9$ ✓

2

ب) $\lim_{x \rightarrow 2^-} [2x-2] = [12-2] = [10^-] = 9$ ✓

ج) $\left[\lim_{x \rightarrow 2^-} (2x-2) \right] = 10$ ✓

د) $\left[\lim_{x \rightarrow 2^+} (2x-2) \right] = 10$ ✓

2

الف) $\lim_{x \rightarrow 2^-} \frac{3[x]+1}{x} = \frac{9+1}{2} = \frac{10}{2}$ ✓

3

ب) $\lim_{x \rightarrow 2^-} \left[\frac{3x+1}{x} \right] = [5^-] = 4$ ✓

ج) $\left[\lim_{x \rightarrow 2^-} \frac{3x+1}{x} \right] = 5$ ✓

2

د) $\left[\lim_{x \rightarrow 2^+} \frac{3x+1}{x} \right] = 5$ ✓

$$\text{ج) } \lim_{x \rightarrow 1} \frac{x-2}{x^2-4x+4} = \frac{x-2}{(x-1)(x-2)} = \lim_{x \rightarrow 1} \frac{1}{x-1}$$

$\lim_{x \rightarrow 1^+} \frac{1}{x-1} = \frac{1}{0^+} = +\infty$ K
 $\lim_{x \rightarrow 1^-} \frac{1}{x-1} = \frac{1}{0^-} = -\infty$

$$\text{د) } \lim_{x \rightarrow 2} \frac{x-3}{x^2-4x+4} = \frac{x-3}{(x-2)^2}$$

$\xrightarrow{2^+} \frac{-1}{0^+} = -\infty$
 $\xrightarrow{2^-} \frac{-1}{0^-} = -\infty$ ✓

(1, 0)

$$\text{الف) } \lim_{x \rightarrow -3} [-3x] - [2x]$$

$\xrightarrow{-3^+} [9^-] - [-12^+] = 1 + 12 = 13$
 $\xrightarrow{-3^-} [4^+] - [-12^-] = 4 + 12 = 16$ ✓

(1)

$$\text{ب) } \lim_{x \rightarrow (-\frac{1}{4})^-} \left\lceil \frac{-1}{x^2} \right\rceil = \left\lceil \frac{1}{\frac{1}{14}} \right\rceil = \lceil 14 \rceil = 14$$

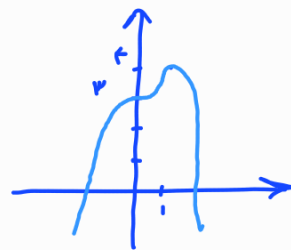
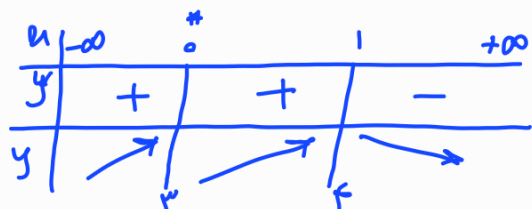
$$\text{ج) } \lim_{x \rightarrow (-\frac{1}{4})^-} \left\lfloor \frac{-1}{x^2} \right\rfloor = -14$$

$$x < -\frac{1}{4} \rightarrow x^2 > \frac{1}{14} \rightarrow -x^2 < -\frac{1}{14} \rightarrow -\frac{1}{x^2} > -14$$

الف) $\lim_{x \rightarrow 1} [f(x)^r - f(x)^f + r] \rightarrow \lim_{x \rightarrow 1} [r^-] = r$

سؤال 4

$y = f(x)^r - f(x)^f + r \rightarrow y' = r f(x)^{r-1} - f(x)^f \xrightarrow{y'=0} r f(x)^{r-1} (1-f(x)) = 0 \rightarrow \begin{cases} x=0 \\ x=1 \end{cases}$



ب) $\lim_{x \rightarrow 0} [f(x)^r - f(x)^f + r] = \begin{cases} \lim_{x \rightarrow 0^+} [r^+] = r \\ \lim_{x \rightarrow 0^-} [r^-] = r \end{cases}$

سؤال 17

الف) $\lim_{x \rightarrow r} \frac{|x^r - 1|}{x^r - r}$ $\frac{x}{x^r - 1} \quad - \quad +$

$\lim_{x \rightarrow r^+} \frac{|x^r - 1|}{x^r - r} = \lim_{x \rightarrow r^+} \frac{x^r - 1}{x^r - r} = \lim_{x \rightarrow r^+} \frac{(x-r)(x^r + rx + r^2)}{(x-r)(x+r)} = \frac{1r}{r} = r$

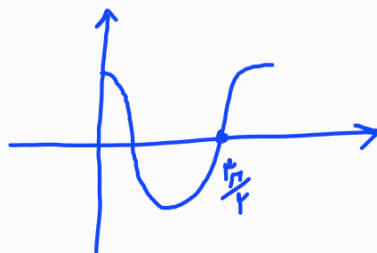
$\lim_{x \rightarrow r^-} \frac{|x^r - 1|}{x^r - r} = \lim_{x \rightarrow r^-} \frac{-(x^r - 1)}{x^r - r} = \lim_{x \rightarrow r^-} \frac{-(x-r)(x^r + rx + r^2)}{(x-r)(x+r)} = \frac{-1r}{r} = -r$

ب) $\lim_{x \rightarrow \pi} \frac{|\sin x|}{\sin 2x}$ $\frac{x}{\sin x} \quad + \quad -$

$\lim_{x \rightarrow \pi^+} \frac{|\sin x|}{\sin 2x} = \lim_{x \rightarrow \pi^+} \frac{-\sin x}{2 \sin x \cos x} = -\frac{1}{2}$

$\lim_{x \rightarrow \pi^-} \frac{|\sin x|}{\sin 2x} = \lim_{x \rightarrow \pi^-} \frac{\sin x}{2 \sin x \cos x} = -\frac{1}{2}$

الف) $\lim_{x \rightarrow \frac{\pi}{4}} \sqrt{\cos x} \rightarrow \begin{cases} \lim_{x \rightarrow \frac{\pi}{4}^+} \sqrt{\cos x} = \lim_{x \rightarrow \frac{\pi}{4}^+} \sqrt{0^+} = 0 \\ \lim_{x \rightarrow \frac{\pi}{4}^-} \sqrt{\cos x} = \lim_{x \rightarrow \frac{\pi}{4}^-} \sqrt{0^-} = \text{تعریف نشده} \end{cases}$



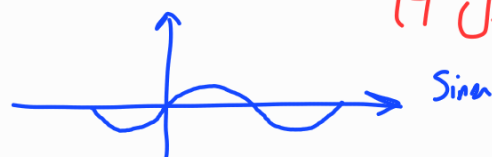
سوال ۱۸

ب) $\lim_{x \rightarrow 0} \sqrt{x - 2\sqrt{x}} = \text{تعریف نشده}$ $x \geq 0$ (I)

$x - 2\sqrt{x} \geq 0 \rightarrow \sqrt{x}(\sqrt{x} - 2) \geq 0 \rightarrow \frac{0}{+|-|+} \rightarrow x \geq 4 \leq x \leq 0$ (II)

دامنه: $x \geq 4 \leq x \leq 0$: $(I) \cap (II) \rightarrow x \geq 4 \leq x = 0$: هیچ نقطه ای در این طرف تعریف نشده.

الف) $\lim_{x \rightarrow 0^+} \frac{(-1)^{[\sin x]}}{x^2 - x^2} = \lim_{x \rightarrow 0^+} \frac{(-1)^{[0^+]}}{x^2(x-1)} = \lim_{x \rightarrow 0^+} \frac{1}{0^-} = -\infty$



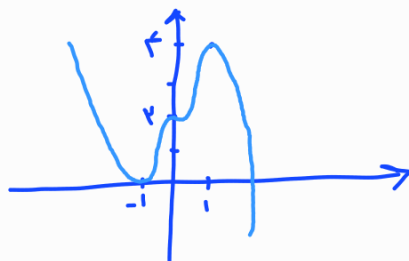
سوال ۱۹

ب) $\lim_{x \rightarrow 2} \frac{x^2 - [x^2]}{x - [x]} = \begin{cases} \lim_{x \rightarrow 2^+} \frac{x^2 - [x^2]}{x - [x]} = \lim_{x \rightarrow 2^+} \frac{x^2 - 4}{x - 2} = \lim_{x \rightarrow 2^+} x + 2 = 4 \\ \lim_{x \rightarrow 2^-} \frac{x^2 - [x^2]}{x - [x]} = \lim_{x \rightarrow 2^-} \frac{x^2 - 3}{x - 1} = \frac{4 - 3}{2 - 1} = 1 \end{cases}$

الف) $y = 5x^3 - 3x^2 + 2$

$y' = 15x^2 - 6x \xrightarrow{y'=0} 15x^2(1-x) = 0 \rightarrow 15x^2(1-x)(1+x) = 0 \rightarrow \begin{cases} x = 0 \\ x = 1 \\ x = -1 \end{cases}$

x	-∞	-1	0	1	+∞
y'	-	+	+	-	
y	↘	↗	↗	↘	



سوال ۱۰

ب) $y = x^4 - 2x^2 + 3$

$y' = 4x^3 - 4x \xrightarrow{y'=0} 4x(x^2 - 1) = 0 \rightarrow 4x(x-1)(x+1) = 0 \rightarrow \begin{cases} x = 0 \\ x = 1 \\ x = -1 \end{cases}$

x	-∞	-1	0	1	+∞
y'	-	+	-	+	
y	↘	↗	↘	↗	

