۱۷,۷۵

سمیرالعلیمی

فاطمه شُعبان

تمرین ۱۷

۸

1) $8\cos x - \tan^2 x = 1$ $[0, 2\pi]$ ۲

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$\hookrightarrow 8\cos x = 1 + \tan^2 x \Rightarrow 8\cos x = \frac{1}{\cos^2 x} \to \cos^3 x = \frac{1}{8}$

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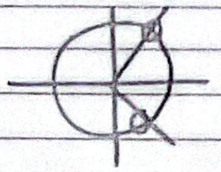
$\to \cos x = \frac{1}{2} \to$



$\to$ $\boxed{\text{۲ جواب}}$ ✓

۱۱

2) $5\sin^2(x) + 2\cos(3x) = -2$ $[-\pi, \pi]$

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$\underbrace{5\sin^2(x)}_{\frac{1-\cos 2x}{2}} + 2\underbrace{\cos(3x)}_{4\cos^3 x - 3\cos x} = -2$

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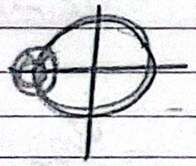
$16\cos^3 x - 10\cos^2 x - 12\cos x + 14 = 0 \to 8\cos^3 x - 5\cos^2 x - 6\cos x + 7 = 0$

$\xrightarrow{\div} \cos x = -1 \longrightarrow$



$\to x = -\pi, \pi$

۱۴

$\boxed{\text{۲ جواب}}$ ✓

۱۵

3) $2\sin(x)\cos(2x) + \sin(x) = 1$ $[0, 2\pi]$ ۱,۵

۱۶

$2\sin x\,(\underbrace{\cos(2x)}) + \sin(x) - 1 = 0 \to 2\sin x(1 - 2\sin^2 x) + \sin(x) - 1 = 0$

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$\sin^2 x = \frac{1-\cos 2x}{2} \to \cos 2x = 1 - 2\sin^2 x$

۱۸

$\leftarrow -4\sin^3 x + 2\sin x + \sin x - 1 = 0$

$\underbrace{-4\sin^3 x + 3\sin x}_{\sin 3x} = 1$

۱۹

$\hookrightarrow \sin 3x = 1 \to 3x = \frac{\pi}{2} \to \boxed{x = \frac{\pi}{6}}$

طبق فرمول

۲۰

$\sin 3x = 1 \to 3x = 2k\pi + \frac{\pi}{2} \to x = \frac{2k\pi}{3} + \frac{\pi}{6} \xrightarrow{0 \le x \le 2\pi}$ ادامه راه‌حل؟

$k = 0 \to x = \frac{\pi}{6}$, $k = 1 \to x = \frac{5\pi}{6}$, $k = 2 \to x = \frac{9\pi}{6}$

$S = \frac{\pi}{6} + \frac{5\pi}{6} + \frac{9\pi}{6} = \frac{15\pi}{6} = \boxed{\frac{5\pi}{2}}$

◆ روز جهانی استاندارد

۴) $\dfrac{1}{\sin\left(\frac{\pi+4x}{2}\right)}+\dfrac{1}{\cos\left(\frac{\pi+8x}{2}\right)}=0$ $\tan 2\alpha$ کدام؟ ، $\alpha \in [0,\pi]$

②

$$\dfrac{1}{\underset{\sin\left(\frac{\pi}{2}+2x\right)}{\cos 2x}}+\dfrac{1}{\underset{\cos\left(\frac{\pi}{2}+4x\right)}{-\sin 4x}}=\dfrac{1}{\cos 2x}+\dfrac{1}{-2\sin 2x\cos 2x}$$

$$\dfrac{-2\sin 2x+1}{-2\sin 2x\cos 2x}=0\longrightarrow -2\sin 2x+1=0\rightarrow \sin 2x=\dfrac{1}{2}$$

$$\hookrightarrow 2x=\dfrac{\pi}{6}\rightarrow x=\dfrac{\pi}{12}$$

$$2x=\dfrac{5\pi}{6}\rightarrow x=\dfrac{5\pi}{12}\qquad\rightarrow\dfrac{5\pi-\pi}{12}=\dfrac{\pi}{3}$$

$$\rightarrow \tan 2\times\dfrac{\pi}{3}=\tan\dfrac{2\pi}{3}=\boxed{-\sqrt{3}}$$ ✓

۵) $m(\cos x-\sin x)-3\sqrt{6}\sin(2x)=\sqrt{6}$, $\cos\left(x+\frac{\pi}{4}\right)=\frac{1}{\sqrt{3}}$

$\rightarrow m=?$

②

$$\hookrightarrow \cos\left(x+\frac{\pi}{4}\right)=\frac{1}{\sqrt{3}}\rightarrow \cos x\cos\frac{\pi}{4}-\sin x\sin\frac{\pi}{4}=\frac{1}{\sqrt{3}}$$

$$\frac{\sqrt{2}}{2}(\cos x-\sin x)=\frac{1}{\sqrt{3}}$$

$$\rightarrow \underline{\cos x-\sin x}=\frac{1}{\sqrt{3}}\times\frac{2}{\sqrt{2}}=\frac{2}{\sqrt{6}}$$

$$*\ (\cos x-\sin x)^2=\underbrace{\cos^2 x+\sin^2 x}_{1}-2\sin x\cos x=\frac{4}{6}=\frac{2}{3}$$

$$\rightarrow \underline{2\sin x\cos x}=\frac{1}{3}$$ ✓

صفحه

$m(\cos x - \sin x) - ۲\sqrt{۶}\sin(۲x) = \sqrt{۶} \longrightarrow$

$\frac{۲}{\sqrt{۶}}m - \cancel{۲}\sqrt{۶}\times\frac{۱}{\cancel{۲}} = \sqrt{۶} \longrightarrow \frac{۲}{\sqrt{۶}}m = ۲\sqrt{۶} \to m = \cancel{۲}\sqrt{۶}\times\frac{\sqrt{۶}}{\cancel{۲}}$

$\boxed{m = ۶}$ ✓

۶) $\sin(x+\frac{\pi}{۶})\cos(x-\frac{\pi}{۳}) = ۱ \to$ $[۰, ۲\pi]$

↓ ۱,۲۵

$(\sin x\cos\frac{\pi}{۶} + \sin\frac{\pi}{۶}\cos x)(\cos x\cos\frac{\pi}{۳} - \sin x\sin\frac{\pi}{۳}) = ۱$

$(\frac{\sqrt{۳}}{۲}\sin x + \frac{۱}{۲}\cos x)(\frac{۱}{۲}\cos x - \frac{\sqrt{۳}}{۲}\sin x) = ۱$

$\cancel{\frac{\sqrt{۳}}{۴}\sin x\cos x} - \overset{\frac{۳}{۴}(۱-\cos^۲ x)}{\underbrace{\frac{۳}{۴}\sin^۲ x}} + \frac{۱}{۴}\cos^۲ x - \cancel{\frac{\sqrt{۳}}{۴}\sin x\cos x} = ۱$

$\frac{۱}{۴}\cos^۲ x - \frac{۳}{۴} + \frac{۳}{۴}\cos^۲ x = ۱ \to \cos^۲ x = \frac{۷}{۴}$ $\alpha < ۱$ ✗ غ ق ق

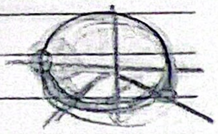
$\boxed{\text{۵ جواب}}$

۷) $\sin x + \sqrt{۳}\cos x = \sqrt{۲}$ $\xrightarrow{} \tan\alpha = \sqrt{۳}$ $[-\pi, ۲\pi]$

۱,۵

$\frac{۱۵\pi}{۸}$,

$\frac{۱}{|۱|}\sqrt{۳+۱}$ $\sin(x+\frac{\pi}{۳}) = \sqrt{۲}$



$۲\sin(x+\frac{\pi}{۳}) = \sqrt{۲} \to \sin(x+\frac{\pi}{۳}) = \frac{\sqrt{۲}}{۲} \to$

$x+\frac{\pi}{۳} = \frac{\pi}{۴} \to x = \frac{-\pi}{۱۲}$ ✓ $\to \frac{۵\pi}{۱۲} - \frac{\pi}{۱۲} = \boxed{\frac{\pi}{۳}}$

$x+\frac{\pi}{۳} = \frac{۳\pi}{۴} \to x = \frac{۹\pi - ۴\pi}{۱۲} = \frac{۵\pi}{۱۲}$ ✓

$x+\frac{\pi}{۳} =$

Scanned with CamScanner

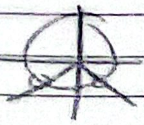
[۰,π]

۸) $۴\sin x \sin(\frac{۳\pi}{۲}-x)=۱$ ۸

۱۵

$\underbrace{\quad\quad}_{-\cos x} \rightarrow -۴\sin x\cos x=۱ \rightarrow -۲\sin ۲x=۱$

$\rightarrow \sin ۲x=-\frac{۱}{۲} \rightarrow$ ۱) $۲x=\frac{۷\pi}{۶} \rightarrow x=\frac{۷\pi}{۱۲}$ ⊕ $\boxed{\frac{۳\pi}{۲}}$ ۹



۲) $۲x=\frac{۱۱\pi}{۶} \rightarrow x=\frac{۱۱\pi}{۱۲}$ ۱۰

۹) ۵ ۱۱

$۱+\cos\alpha\ ۲\cos^۲(\frac{\alpha}{۲}) \rightarrow ۸\cos^۲\alpha\cos^۲(۲\alpha)\cos^۲(۴\alpha)=\frac{۱}{۸}$ ۱۲

$|\cos\alpha\cos۲\alpha\cos۴\alpha|=\frac{۱}{۸} \rightarrow \sin ۸\alpha=۸\sin\alpha\cos\alpha\cos۲\alpha\cos۴\alpha$

$\frac{\sin ۸\alpha}{\sin\alpha}=۱ \rightarrow |\sin ۸\alpha|=|\sin\alpha| \rightarrow \max=\frac{۸\pi}{۹}$ ✓ ۱۳

۱۴

۱۰) ۱۵

~~۱۰) $\tan(۳x)\tan(x)=۱ \rightarrow$~~

[۰,۲π]

✓ ۵

$۳x=\frac{\pi}{۲}-x+k\pi \rightarrow ۴x=\frac{\pi}{۲}+k\pi \rightarrow x=\frac{۹\pi}{۸},\frac{۱۱\pi}{۸},\frac{۱۳\pi}{۱۸},\frac{۱۵\pi}{۸}$

$\rightarrow ۹\ \frac{۴۸\pi}{۸}=۶\pi \rightarrow \boxed{۶\pi}$ ✓

۱۸

۱۹

۲۰

(۶)

$$\cos(x-\frac{\pi}{۳})=\cos(\frac{\pi}{۳}-x)$$

$$\boxed{\frac{\pi}{۳}-x+x+\frac{\pi}{۶}=\frac{\pi}{۲}}\longrightarrow \cos(\frac{\pi}{۳}-x)=\sin(x+\frac{\pi}{۶})$$

$$\sin^{۲}(x+\frac{\pi}{۶})=۱\longrightarrow \sin(x+\frac{\pi}{۶})=\pm۱\longrightarrow x+\frac{\pi}{۶}=k\pi+\frac{\pi}{۲}$$

$$x=k\pi+\frac{\pi}{۳}\xrightarrow{۰\le x\le ۲\pi} k=۰\to x=\frac{\pi}{۳}\ ,\ k=۱\ ,\ x=\frac{۴\pi}{۳}$$

معادله ۲ جواب در بازه دارد

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(۷) عبارت را در $\frac{۱}{۲}$ ضرب می‌کنیم.

$$\frac{۱}{۲}\sin x+\frac{\sqrt{۳}}{۲}\cos x=\frac{\sqrt{۲}}{۲}$$

$$\cos۶۰^\circ\sin x+\sin۶۰^\circ\cos x=\frac{\sqrt{۲}}{۲}\longrightarrow \sin(x+\frac{\pi}{۳})=\frac{\sqrt{۲}}{۲}$$

$$\begin{cases}x+\frac{\pi}{۳}=۲k\pi+\frac{\pi}{۴}\longrightarrow x=۲k\pi-\frac{\pi}{۱۲}\xrightarrow{-\pi\le x\le ۲\pi} k=۰,۱\to x=-\frac{\pi}{۱۲},\frac{۲۳\pi}{۱۲}\\ x+\frac{\pi}{۳}=۲k\pi+\frac{۳\pi}{۴}\longrightarrow x=۲k\pi+\frac{۵\pi}{۱۲}\xrightarrow{-\pi\le x\le ۲\pi} k=۰\to x=\frac{۵\pi}{۱۲}\end{cases}$$

$$S=\frac{۲۳\pi}{۱۲}-\frac{\pi}{۱۲}+\frac{۵\pi}{۱۲}=\frac{۲۷\pi}{۱۲}=\boxed{\frac{۹\pi}{۴}}$$

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(۸)

$$\sin(\frac{۳\pi}{۲}-x)=-\cos x$$

$$۴\sin x(-\cos x)=۱\longrightarrow -۲(۲\sin x\cos x)=۱\longrightarrow \sin۲x=-\frac{۱}{۲}$$

$$۲x=۲k\pi-\frac{\pi}{۶}\ \text{یا}\ ۲k\pi+\frac{۷\pi}{۶}\longrightarrow x=\overset{2}{k\pi}-\frac{\pi}{۱۲}\ \text{یا}\ \overset{1}{k\pi}+\frac{۷\pi}{۱۲}$$

$$\xrightarrow{۰\le x\le ۲\pi}\begin{cases}1\ \ k=۰,۱\longrightarrow x=\frac{۷\pi}{۱۲},\frac{۱۹\pi}{۱۲}\\ 2\ \ k=۱,۲\longrightarrow x=\frac{۱۱\pi}{۱۲},\frac{۲۳\pi}{۱۲}\end{cases}$$

$$S=\frac{۷\pi}{۱۲}+\frac{۱۹\pi}{۱۲}+\frac{۱۱\pi}{۱۲}+\frac{۲۳\pi}{۱۲}=\frac{۶۰\pi}{۱۲}=\boxed{۵\pi}$$