۱۷/۷۵ مرتب تر بنویس!

نام و نام خانوادگی .... پاسخنامه تشریحی تکلیف شماره ۱۷ کلاس .... A

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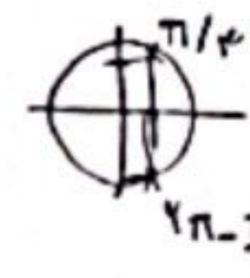
$8\cos x = 1+\tan^2 x \to \frac{1}{\cos^2 x} = 8\cos x$ $\quad \cos x \neq 0$

$\frac{1}{\cos^3 x} = 8 \to \cos x = \frac{1}{2}$



$\to x = 2k\pi \pm \frac{\pi}{3}$

۲ جواب دارد

$x = \frac{5\pi}{3}$ و $x = \frac{\pi}{3}$

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× جواب داره ۲ جواب دارد!

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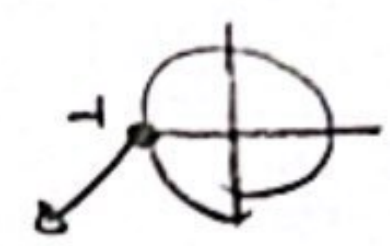
$5\sin^2(x) + 2\cos(4x) = -2$

$2\cos 4x = -2$ $\quad \cos 4x = -1$



اگر $\sin^2 x$ صفر بشه cos جواب داره

اما اگر $\sin^2 x$ ۱ بشه

$5 + 2\cos 4x = -2 \to \cos 4x = -3.5$ جواب نداره ×

$4x = 2k\pi + \pi$

$\boxed{x = \frac{2k\pi}{4} + \frac{\pi}{4}}$ $\to \frac{\pi}{4}, \pi, \frac{-2\pi}{4}+\frac{\pi}{4}, \frac{-\pi}{4}+\frac{\pi}{4}$

باید بین جواب ها اشتراک بگیری!

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$\sin x(2\cos 2x + 1) = 1$

$\cos 2x = \cos^2 x - \sin^2 x$

$\sin x(\cos^2 x - \sin^2 x)$ $\quad 1-\sin^2 x$

$\sin x(2\cos^2 x - 2\sin^2 x + \cos^2 x + \sin^2 x)$

$\cancel{2\sin x}(1-2\sin^2 x) + \sin x = 1 \to -4\sin^3 x + 3\sin x = 1$

$3\sin x - 4\sin^3 x$ $\quad \to \sin 3x = 1$

$3x = 2k\pi + \pi/2$

$\boxed{x = \frac{2k\pi}{3} + \frac{\pi}{6}}$

$x = \frac{\pi}{6}, \frac{2\pi}{3}+\frac{\pi}{6}, \frac{4\pi}{3}+\frac{\pi}{6}$

$= \frac{\pi}{6} + \frac{2\pi}{3}+\frac{\pi}{6} + \frac{4\pi}{3}+\frac{\pi}{6}$

جواب $\frac{5\pi}{2}$ یا $2\pi + \frac{\pi}{2}$

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$\frac{1}{\cos 2x} + \frac{1}{-\sin 4x} = 0$ $\quad \frac{1}{\cos 2x} = \frac{1}{\sin 4x}$ $\quad \sin 4x, \cos 2x \neq 0$

$\tan^2\frac{\pi}{3} = \sqrt{\tan 120^\circ} = -\sqrt{3}$ جواب

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$\sin 4x = \cos 2x \to \sin 4x = \sin(\frac{\pi}{2} - 2x)$

$4x = 2k\pi + \frac{\pi}{2} - 2x \to 6x = 2k\pi + \frac{\pi}{2}$ $\quad \boxed{x = \frac{k\pi}{3} + \frac{\pi}{12}}$

$4x = 2k\pi + \pi - \frac{\pi}{2} + 2x$ $\quad 2x = 2k\pi + \frac{\pi}{2}$

$x = k\pi + \frac{\pi}{4}$ که در این صورت $\cos 2x$ و $\sin 4x$ صفر میشن $\quad 2x = 2k\pi + \pi/2$

$x = \frac{\pi}{12}$ $\quad x = \frac{\pi}{4} + \frac{\pi}{12}$

$x = \frac{\pi}{4} + \frac{\pi}{12} = \frac{5\pi}{12}$

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$m\cos x - m\sin x - 2\sqrt{6}(2\sin x\cos x) = \sqrt{6}$

$m(\cos x - \sin x) - 2\sqrt{6}\sin 2x = \sqrt{6}$ در معادله صدق می‌کند

$\cos(x + \frac{\pi}{4}) = \frac{1}{\sqrt{3}}$

$\cos x\cos\frac{\pi}{4} - \sin x\sin\frac{\pi}{4} = \frac{1}{\sqrt{3}}$

$\frac{1}{\sqrt{2}}(\cos x - \sin x) = \frac{1}{\sqrt{3}}$

$\cos x - \sin x = \frac{\sqrt{2}}{\sqrt{3}}$

$1 - \sin 2x = \frac{2}{3}$

$\sin 2x = \frac{1}{3}$

$\frac{\sqrt{2}}{\sqrt{3}}m - 4\sqrt{6}\times\frac{1}{3} = \sqrt{6}$

$\frac{\sqrt{2}}{\sqrt{3}}m = 2\times\sqrt{6}$

$m = 6$

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مهران

$\sin(x+\frac{\pi}{6})\sin(\underbrace{\frac{\pi}{2}-x+\frac{\pi}{3}}_{\frac{5\pi}{6}-x}) = 1$

$\sin(\frac{\pi}{6}+x)$

$\sin^2(x+\frac{\pi}{6}) = 1$

$\sin(x+\frac{\pi}{6}) = \pm 1$

$\frac{\pi}{2}$, $\frac{3\pi}{2}$

$x+\frac{\pi}{6} = \frac{3\pi}{2}$ $\quad x = \frac{4\pi}{6} = \frac{2\pi}{3}$

$x+\frac{\pi}{6} = \frac{3\pi}{2}$ $\quad x = \frac{8\pi}{6} = \frac{4\pi}{3}$

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$\underbrace{\frac{1}{2}}_{\sin 30}\sin x + \underbrace{\frac{\sqrt{3}}{2}}_{\cos 30}\cos x = \frac{\sqrt{2}}{2}$

$\cos(a+b) = \cos a\cos b - \sin a\sin b$

$\cos(a-b) = \cos a\cos b + \sin a\sin b$

$\cos(30-x) = \cos 45 \quad x = 15°$

$\cos(x-30) = \cos 45 \quad x = -15°$

$x-30 = 45 \quad x = 75$

$\alpha = 75$

$x - \frac{\pi}{6} = 2k\pi \pm \frac{\pi}{4} \rightarrow x = 2k\pi - \frac{\pi}{12} \rightarrow k = 0 , 1 \rightarrow x = -\frac{\pi}{12} , \frac{23}{12}\pi$

$2k\pi + \frac{5\pi}{12} \rightarrow k = 0 \rightarrow x = \frac{5\pi}{12}$

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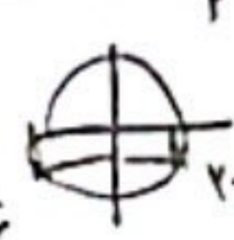
مجموع جواب‌ها

$4\sin x(\underbrace{-\cos x}_{-\frac{\sin 2x}{2}}) = 1$

$2\sin 2x = -1$

$\sin 2x = \frac{-1}{2}$

$\underbrace{\frac{18\pi}{12} + \frac{18\pi}{12}}_{3\pi} + 2\pi = 5\pi$



$\frac{7\pi}{6} = \pi + \frac{\pi}{6}$ $\quad 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$

$2x = \frac{11\pi}{6} \rightarrow x = \frac{11\pi}{12}$

$2x = \frac{7\pi}{6} \rightarrow x = \frac{7\pi}{12}$

$2x = \frac{7\pi}{6} + 2\pi \rightarrow x = \frac{7\pi}{12} + \pi$

$2x = \frac{11\pi}{6} + 2\pi \rightarrow x = 11\pi/12 + \pi$

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$\frac{3\pi}{16}$

$(1+\cancel{\cos 2x})$ $\quad (1 \pm \cos^2 2x - \sin^2 2x)$ $\quad 2\cos^2 4x = \frac{1}{8}$

$\underbrace{\cos^2 x - \sin^2 x}_{2\cos^2 x}$ $\quad 2\cos^2 2x$

$\cos^2 x \cos^2 2x \cos^2 4x = \frac{1}{64}$

$\boxed{x = \frac{3\pi}{16}}$ یا $8x = \frac{3\pi}{2}$

$8x = \frac{\pi}{2}$

$8\sin^2 x$ و

$\frac{1}{4}\cancel{\sin^2 2x \cos^2 2x}\cos^2 4x$

$\frac{1}{4}\sin^2 4x$

$\frac{1}{4}\times\frac{1}{4}\times\frac{1}{4}\sin^2 8x = \frac{1}{64} \rightarrow \sin^2 8x = 1$

دو طرف عبارت در $\sin x$ ضرب می‌شود !!

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مهران $\frac{7\pi}{2}$

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$\tan 3x = \frac{1}{\tan x} \rightarrow \tan 3x = \cot x \rightsquigarrow \tan 3x = \tan(\frac{\pi}{2}-x)$

$3x = 2k\pi + \frac{\pi}{2} - x \longrightarrow 4x = 2k\pi + \pi/2 \rightsquigarrow x = \frac{k\pi}{2} + \frac{\pi}{8}$

$-2\cancel{\frac{\pi}{8}}$ و $\frac{\pi}{2}\cancel{+\frac{\pi}{8}}$ و $\frac{\pi}{8}$, $\frac{3\pi}{2}+\frac{\pi}{8}$

$3x = 2k\pi + \pi - \frac{\pi}{2} + x \quad 2x = 2k\pi + \frac{\pi}{2}$

$x = k\pi + \frac{\pi}{4} - 2$ $\quad \pi + \frac{\pi}{4}$

k

$2\pi + \frac{\pi}{4} + \pi + \frac{\pi}{4} = 3\pi + \frac{\pi}{2} = \frac{7\pi}{2}$

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$$\cos 3x = 2\cos^2\frac{3x}{2} - 1$$ (فرمول طلایی)

$$5\sin^2 x + 2\left(2\cos^2\frac{3x}{2} - 1\right) = -2 \longrightarrow 5\sin^2 x + 4\cos^2\frac{3x}{2} \cancel{-2} = \cancel{-2}$$

حاصل جمع دو عدد نامنفی صفر است پس هر دو صفر بوده اند!

$$5\sin^2 x = 0 \longrightarrow \sin x = 0 \longrightarrow x = k\pi \xrightarrow{-\pi \le x \le \pi} x = \pi \text{ یا } -\pi \text{ یا } 0$$

$$4\cos^2\frac{3x}{2} = 0 \longrightarrow \cos\frac{3x}{2} = 0 \longrightarrow \frac{3x}{2} = k\pi + \frac{\pi}{2} \longrightarrow x = \frac{2k\pi + \pi}{3}$$

$$\xrightarrow{x \le \pi} x = -\pi \text{ یا } -\frac{\pi}{3} \text{ یا } \frac{\pi}{3} \text{ یا } \pi$$

$$\boxed{\text{اشتراک دو مجموعه جواب } \{-\pi, \pi\} \text{ است}}$$

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$$\tan 3x = \frac{1}{\tan x} = \cot x \longrightarrow \tan 3x = \tan\left(\frac{\pi}{2} - x\right)$$

$$3x = k\pi + \frac{\pi}{2} - x \longrightarrow 4x = k\pi + \frac{\pi}{2} \longrightarrow x = \frac{k\pi}{4} + \frac{\pi}{8}$$

k	4	5	6	7
x	$\frac{9\pi}{8}$	$\frac{11\pi}{8}$	$\frac{13\pi}{8}$	$\frac{15\pi}{8}$

$$S = \left(\frac{9 + 11 + 13 + 15}{8}\right)\pi = \frac{48\pi}{8} = \boxed{6\pi}$$

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۷ عبارت را در $\frac{1}{2}$ ضرب می کنیم.

$$\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x = \frac{\sqrt{2}}{2}$$

$$\cos 60^\circ \sin x + \sin 60^\circ \cos x = \frac{\sqrt{2}}{2} \longrightarrow \sin\left(x + \frac{\pi}{3}\right) = \frac{\sqrt{2}}{2}$$

$$\begin{cases} x + \frac{\pi}{3} = 2k\pi + \frac{\pi}{4} \longrightarrow x = 2k\pi - \frac{\pi}{12} \xrightarrow{-\pi \le x \le 2\pi} k = 0, 1 \longrightarrow x = -\frac{\pi}{12}, \frac{23\pi}{12} \\ x + \frac{\pi}{3} = 2k\pi + \frac{3\pi}{4} \longrightarrow x = 2k\pi + \frac{5\pi}{12} \xrightarrow{-\pi \le x \le 2\pi} k = 0 \longrightarrow x = \frac{5\pi}{12} \end{cases}$$

$$S = \frac{23\pi}{12} - \frac{\pi}{12} + \frac{5\pi}{12} = \frac{27\pi}{12} = \boxed{\frac{9\pi}{4}}$$

$1+\cos 2x = 2\cos^2 x$ *

$$\begin{cases} 1+\cos 2\alpha = 2\cos^2\alpha \\ 1+\cos 4\alpha = 2\cos^2 2\alpha \\ 1+\cos 8\alpha = 2\cos^2 4\alpha \end{cases} \xrightarrow{?} 8\cos^2\alpha\, \cos^2 2\alpha\, \cos^2 4\alpha = \frac{1}{8}$$

$$\cos^2\alpha\, \cos^2 2\alpha\, \cos^2 4\alpha = \frac{1}{64} \xrightarrow{\sqrt{\ }} \cos\alpha \cos 2\alpha \cos 4\alpha = \frac{\pm 1}{8} \xrightarrow[\sin\alpha \neq 0]{\times \sin\alpha \ *}$$

$$\underbrace{(\sin\alpha \cos\alpha)}_{\sin 2\alpha} \cos 2\alpha \cos 4\alpha = \pm\frac{\sin\alpha}{8} \longrightarrow \frac{1}{2}\underbrace{(\sin 2\alpha \cos 2\alpha)}_{\frac{1}{2}\sin 4\alpha} \cos 4\alpha = \pm\frac{\sin\alpha}{8}$$

$$\frac{1}{2}\times\frac{1}{2}\underbrace{(\sin 4\alpha \cos 4\alpha)}_{\sin 8\alpha} = \frac{\pm\sin\alpha}{8} \longrightarrow \frac{1}{\cancel{8}}\sin 8\alpha = \pm\frac{\sin\alpha}{\cancel{8}}$$

$$\sin 8\alpha = \pm\sin\alpha \longrightarrow \begin{cases} 8\alpha = 2k\pi + \alpha \ \text{یا}\ 2k\pi + \pi - \alpha \\ 8\alpha = 2k\pi - \alpha \ \text{یا}\ 2k\pi + \pi + \alpha \end{cases}$$

$$\alpha = \frac{2k\pi}{7} \xrightarrow{\max A} k=3 \longrightarrow x = \frac{6\pi}{7}$$

$$\alpha = \frac{2k\pi}{9} \xrightarrow{\max A} k=4 \longrightarrow x = \frac{8\pi}{9}$$

$$\alpha = \frac{(2k+1)\pi}{7} \xrightarrow{\max A} k=2 \longrightarrow x = \frac{5\pi}{7}$$

$$\alpha = \frac{(2k+1)\pi}{9} \xrightarrow{\max A} k=3 \longrightarrow x = \frac{7\pi}{9}$$

$$\longrightarrow \boxed{\max A = \frac{8\pi}{9}}$$

* A نمی‌تواند برابر خود π باشد چون طبق * $\sin\alpha$ را مخالف صفر در نظر گرفته ایم!