نگین نکویی — دوازدهم ریاضی ۵ — تکلیف ۱۷

۱-

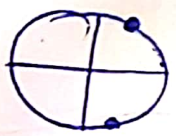
$8\cos x - \tan^2 x = 1$

$8\cos x = 1 + \tan^2 x = \frac{1}{\cos^2 x}$

$8\cos x - \frac{1}{\cos^2 x} = 0 \qquad \cos x \neq 0$

$8\cos^3 x - 1 = 0$

$8\cos^3 x = 1 \rightarrow \cos^3 x = \frac{1}{8} \rightarrow \cos x = \frac{1}{2}$



$x = 2k\pi \pm \frac{\pi}{3}$ → $\frac{\pi}{3}$ و $\frac{5\pi}{3}$ جواب

۲-

$\cos 3\alpha = 4\cos^3\alpha - 3\cos\alpha$ فرمول کانتری

$5(1-\cos^2 x) + 2(4\cos^3 x - 3\cos x) = -2$

$8\cos^3 x - 5\cos^2 x - 6\cos x + 7 = 0 \rightarrow \cos x = -1$

با تقسیم به هورنر

8	-5	-6	+7
↓	-8	13	-7
8	-13	7	0

$(\cos x + 1)(8\cos^2 x - 13\cos x + 7)$

$\cos x = -1$

$\Delta = b^2 - 4ac = 169 - 4\times 8\times 7 < 0$ جواب ندارد

$\cos x = -1 \rightarrow x = 2k\pi + \pi$

$-\pi$ و π

۳-

$2\sin x(1 - 2\sin^2 x) + \sin x = 1$

$\underbrace{-4\sin^3 x + 3\sin x}_{\sin 3x} - 1 = 0 \rightarrow \sin 3x = +1$

$3x = 2k\pi + \frac{\pi}{2}$

$x = \frac{2k\pi}{3} + \frac{\pi}{6}$ → $\frac{\pi}{6}$, $\frac{5\pi}{6}$, $\frac{3\pi}{2}$

$\frac{\pi}{6} + \frac{5\pi}{6} + \frac{3\pi}{2} = \frac{5\pi}{2}$

$$\frac{1}{\sin(\frac{\pi}{2}+2x)} + \frac{1}{\cos(\frac{\pi}{2}+2x)} = 0$$

$$\cos 2x \qquad -\sin 2x$$

$$\frac{2\sin 2x - 1}{\sin 2x \cos 2x} = 0 \quad \rightarrow 2\sin 2x - 1 = 0$$

$$\sin 2x = \frac{1}{2} \rightarrow 2x = 2k\pi + \frac{\pi}{6} \rightarrow x = k\pi + \frac{\pi}{12}$$

$$\hookrightarrow 2x = 2k\pi + \frac{5\pi}{6} \rightarrow x = k\pi + \frac{5\pi}{12}$$

$$\tan\frac{2\pi}{3} = -\sqrt{3} \qquad \alpha = \frac{2\pi}{3} = \frac{\pi}{3}$$

ب -

$$M(\cos x - \sin x) - 2\sqrt{6}\sin(2x) = \sqrt{6}$$

$$\cos x - \sin x = -(\sin x - \cos x) = \sqrt{2}\sin(\frac{\pi}{2} - x) = \sqrt{2}\cos(\frac{\pi}{2} + x) \Rightarrow \sqrt{2} \times \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\cos(\underbrace{x + \frac{\pi}{4}}_{\alpha}) = \frac{1}{\sqrt{2}} \rightarrow \cos\alpha = \frac{1}{\sqrt{2}} \rightarrow \cos 2\alpha = 2\cos^2\alpha - 1 \Rightarrow \cos 2\alpha = 2 \times \frac{1}{2} - 1 = \frac{-1}{2}$$

$$\cos(2x + \frac{\pi}{2}) = -\frac{1}{2}$$

$$-\sin 2x = -\frac{1}{2} \rightarrow \sin 2x = \frac{1}{2}$$

$$\frac{M}{\sqrt{2}} \times \frac{\sqrt{2}}{2} - 2 \times \sqrt{6} \times \frac{1}{2} = \sqrt{6} \rightarrow$$

$$\frac{M\sqrt{2}}{2\sqrt{2}} = 2\sqrt{6} \rightarrow M\sqrt{2} = 2\sqrt{12} \rightarrow M = 6$$

ج -

$$\sin(x + \frac{\pi}{6}).\cos(x - \frac{\pi}{12}) = 1$$

د- ~~$\sin(\frac{\pi}{2} + x - \frac{\pi}{6}) = -\cos(\frac{\pi}{2})$~~

$$\cos(x - \frac{\pi}{12}) = \cos(\frac{\pi}{12} - x) \rightarrow \sin(x + \frac{\pi}{6})\cos(\frac{\pi}{12} - x) = 1$$

$$\sin^2(x + \frac{\pi}{6}) = 1$$

$$\sin(x + \frac{\pi}{6}) = \pm 1$$

$$x + \frac{\pi}{6} = 2k\pi + \frac{\pi}{2} \rightarrow x = 2k\pi + \frac{\pi}{3}$$

$$x + \frac{\pi}{6} = 2k\pi + \frac{3\pi}{2} \rightarrow x = 2k\pi + \frac{8\pi}{6}$$

الحلول $\frac{\pi}{3}$ ، $\frac{8\pi}{6}$

$$\sin x + \frac{\sin\frac{\pi}{12}}{\cos\frac{\pi}{12}}\cos x = \sqrt{2}$$

$$\frac{\sin x\cos\frac{\pi}{12} + \sin\frac{\pi}{6}\cos x}{\cos\frac{\pi}{12}} = \sqrt{2}$$

$\frac{1}{2}$

$$\sin\left(x+\frac{\pi}{6}\right) = \frac{\sqrt{2}}{2} \rightarrow x+\frac{\pi}{6} = 2k\pi + \frac{\pi}{4} \rightarrow x = 2k\pi - \frac{\pi}{12}$$

$$x+\frac{\pi}{6} = 2k\pi + \frac{3\pi}{4} \rightarrow x = 2k\pi + \frac{7\pi}{12}$$

$$-\frac{\pi}{12} + \frac{7\pi}{12} + \frac{23\pi}{12} = \frac{9\pi}{4}$$

$$-\frac{\pi}{12},\ \frac{7\pi}{12},\ \frac{23\pi}{12}$$

-۸

$$-4\sin x\cos x = 1 \rightarrow -2\times\underbrace{2\sin x\cos x}_{\sin 2x} = 1$$

$$\sin 2x = -\frac{1}{2}$$

$$\sin\left(\frac{3\pi}{2} - x\right) = -\cos x$$

$$\rightarrow 2x = 2k\pi + \frac{7\pi}{6} \rightarrow x = k\pi + \frac{7\pi}{12}$$

$$2x = 2k\pi + \frac{11\pi}{6} \rightarrow x = k\pi - \frac{\pi}{12}$$

$$\frac{11\pi}{12}\quad \frac{23\pi}{12}\quad \frac{7\pi}{12}\quad \frac{19\pi}{12}$$

5π

-۹

$$(1+\cos 2\alpha)(1+\cos 4\alpha)(1+\cos 8\alpha) = \frac{1}{8}$$

$2\cos^2 2\alpha \quad 2\cos^2 2\alpha \quad 2\cos^2 4\alpha$

$$\cos^2\alpha = \frac{1+\cos 2\alpha}{2}$$

$$\max \rightarrow \frac{8\pi}{9}$$

$$\times\sin^2\alpha \rightarrow \frac{8}{8}\sin^2\alpha\cos^2\alpha\cos^2 2\alpha\cos^2 4\alpha \quad \times 8$$

$$\sin x\cos\alpha = \frac{1}{2}\sin 2\alpha$$

$\frac{1}{4}\sin^2 2\alpha$ $\quad \frac{1}{16}\sin^2 4\alpha$ $\quad \frac{1}{4}\sin^2 8\alpha$

$$\frac{1}{8}\cdot\frac{\sin^2 8\alpha}{\sin^2\alpha} = \frac{1}{8}$$

$$\sin^2\cos^2\alpha = \frac{1}{4}\sin^2 2\alpha$$

$$\sin 8\alpha = -\sin\alpha$$

$$\sin 8\alpha = \sin(-\alpha)$$

$$8\alpha = 2k\pi - \alpha \rightarrow \alpha = \frac{2k\pi}{9}$$

$$8\alpha = 2k\pi + \pi + \alpha \rightarrow \alpha = \frac{2k\pi}{7} + \frac{\pi}{7}$$

$$\sin^2 8\alpha = \sin^2 a$$

$$\sin 8\alpha = \pm\sin a \rightarrow \sin 8\alpha = \sin a$$

$$8\alpha = 2k\pi + a \rightarrow \alpha = \frac{2k\pi}{7}$$

$$8\alpha = 2k\pi + \pi - a \rightarrow \alpha = \frac{2k\pi}{9} + \frac{\pi}{9}$$

$$\frac{\pi}{7},\ \frac{\pi}{9},\ \frac{4\pi}{7},\ \frac{2\pi}{7},\ \frac{4\pi}{9},\ \frac{3\pi}{4},\ \frac{10\pi}{9},\ \frac{8\pi}{9},\ \frac{6\pi}{7},\ \frac{5\pi}{4},\ \frac{12\pi}{9},\ \frac{\pi}{9},\ \frac{5\pi}{7},\ \frac{9\pi}{9}$$

$\tan(\frac{\pi}{2} - x)\tan(x) = 1 \Rightarrow \tan x = \frac{1}{\tan(\frac{\pi}{2} - x)}$

$\tan(\frac{\pi}{2} - x) = \frac{1}{\tan(x)}$

$\cot(\frac{\pi}{2} - x) = \tan(\frac{\pi}{2} - x)$

$\frac{\pi}{2} - x = k\pi + \frac{\pi}{2} - x$

$x = k\pi + \frac{\pi}{2} \rightarrow x = \frac{k\pi}{2} + \frac{\pi}{4}$

$\frac{9\pi}{4}$

$\frac{11\pi}{4}$

$\frac{13\pi}{4}$

$\frac{15\pi}{4}$