نگین نکویی — ۱۹/۱۷/۵ — دوازدهم ریاضی — تکلیف ۱۷

۱-

$8\cos x - \tan^2 x = 1$

$8\cos x = \cancel{1+\tan^2 x} = \dfrac{1}{\cos^2 x}$

$8\cos x - \dfrac{1}{\cos^2 x} = 0 \qquad \cos x \neq 0$

$8\cos^3 x - 1 = 0$

$8\cos^3 x = 1 \rightarrow \cos^3 x = \dfrac{1}{8} \rightarrow \cos x = \dfrac{1}{2}$

$x = 2k\pi \pm \dfrac{\pi}{3} \rightarrow \dfrac{\pi}{3} , \dfrac{5\pi}{3}$ جواب

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۲-

$\cos 3\alpha = 4\cos^3\alpha - 3\cos\alpha$ فرمول کسینوسی

$5(1-\cancel{\cos}^2 x) + 2(4\cos^3 x - 3\cos x) = -2$

$8\cos^3 x - 5\cos^2 x - 6\cos x + 7 = 0 \rightarrow \cos x = -1$

با تقسیم بر هورنر

8	-5	-6	$+7$
$\downarrow$	-8	13	-7
8	-13	7	0

$(\cos x + 1)(8\cos^2 x - 13\cos x + 7)$

$\cos x = -1$

$\Delta = b^2 - 4ac = 169 - 4\times 8\times 7 < 0$ جواب ندارد

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$\cos x = -1 \rightarrow x = 2k\pi + \pi$

$-\pi , \pi$

۳-

$2\sin x(1-2\sin^2 x) + \sin x = 1 \rightarrow$

$\underbrace{-4\sin^3 x + 3\sin x}_{\sin 3x} - 1 = 0$

$\sin 3x = +1$

$3x = 2k\pi + \dfrac{\pi}{2}$

$x = \dfrac{2k\pi}{3} + \dfrac{\pi}{6} \rightarrow \dfrac{\pi}{6} , \dfrac{5\pi}{6} , \dfrac{3\pi}{2}$

$\dfrac{\pi}{6} + \dfrac{5\pi}{6} + \dfrac{3\pi}{2} = \dfrac{5\pi}{2}$

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$$\frac{1}{\cancel{\sin(\frac{\pi}{2}+2x)}} + \frac{1}{\cancel{\cos(\frac{\pi}{2}+4x)}} = 0$$

$\cos 2x$ $\qquad$ $-\sin 4x$

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$$\frac{2\sin 2x - 1}{2\sin 2x\cos 2x} = 0 \quad \to 2\sin 2x - 1 = 0$$

$\sin 2x = \frac{1}{2} \to 2x = 2k\pi + \frac{\pi}{6} \to x = k\pi + \frac{\pi}{12}$

$\hookrightarrow 2x = 2k\pi + \frac{5\pi}{6} \to x = k\pi + \frac{5\pi}{12}$

$\tan\frac{2\pi}{3} = -\sqrt{3}$ $\qquad$ $\alpha = \frac{4\pi}{12} = \frac{\pi}{3}$

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۵ -

$$m(\cos x - \sin x) - 3\sqrt{6}\sin(2x) = \sqrt{6}$$

$\cos x - \sin x = -(\sin x - \cos x) = \sqrt{2}\sin(\frac{\pi}{4} - x) = \sqrt{2}\cos(\frac{\pi}{4} + x) \Rightarrow \sqrt{2}\times\frac{1}{\sqrt{3}} = \frac{\sqrt{2}}{\sqrt{3}}$

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$\cos(\underbrace{x + \frac{\pi}{4}}_{\alpha}) = \frac{1}{\sqrt{3}} \to \cos\alpha = \frac{1}{\sqrt{3}} \to \cos 2\alpha = 2\cos^2\alpha - 1 \Rightarrow \cos 2\alpha = 2\times\frac{1}{3} - 1 = -\frac{1}{3}$

$\cos(\underbrace{2x + \frac{\pi}{2}}) = -\frac{1}{3}$

$-\sin 2x = -\frac{1}{3} \to \sin 2x = \frac{1}{3}$

$m\times\frac{\sqrt{2}}{\sqrt{3}} - \cancel{3}\sqrt{6}\times\frac{1}{\cancel{3}} = \sqrt{6} \to$

$\frac{m\sqrt{2}}{\sqrt{3}} = 2\sqrt{6} \to m\sqrt{2} = 2\sqrt{18} \to m = 6$

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۶ -

$$\sin(x + \frac{\pi}{6})\cdot\cos(x - \frac{\pi}{3}) = 1$$

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یا ~~$\sin(\frac{\pi}{2} + x - \frac{\pi}{3}) = (-\cos \ldots)$~~

$\cos(x - \frac{\pi}{3}) = \cos(\frac{\pi}{3} - x) \to \sin(x + \frac{\pi}{6})\cos(\frac{\pi}{3} - x) = 1$ → جمعشان ۹۰ درجه است

$\sin^2(x + \frac{\pi}{6}) = 1$

$\sin(x + \frac{\pi}{6}) = \pm 1$

$x + \frac{\pi}{6} = 2k\pi + \frac{\pi}{2} \to x = 2k\pi + \frac{\pi}{3}$

$x + \frac{\pi}{6} = 2k\pi + \frac{3\pi}{2} \to x = 2k\pi + \frac{8\pi}{6}$

$\frac{\pi}{3}, \frac{8\pi}{6}$ جواب ۲

۷-

$$\sin x + \frac{\sin\frac{\pi}{3}}{\cos\frac{\pi}{3}}\cos x = \sqrt{3}$$

$$\frac{\sin x\cos\frac{\pi}{3} + \sin\frac{\pi}{3}\cos x}{\underbrace{\cos\frac{\pi}{3}}_{\frac{1}{2}}} = \sqrt{3}$$

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$$\sin(x+\frac{\pi}{3}) = \frac{\sqrt{3}}{2} \rightarrow x+\frac{\pi}{3} = 2k\pi+\frac{\pi}{4} \rightarrow x = 2k\pi-\frac{\pi}{12}$$

$$x+\frac{\pi}{3} = 2k\pi+\pi-\frac{\pi}{4} \;\left(\frac{3\pi}{4}\right) \rightarrow x = 2k\pi+\frac{5\pi}{12}$$

$$-\frac{\pi}{12}+\frac{5\pi}{12}+\frac{23\pi}{12} = \frac{9\pi}{4}$$

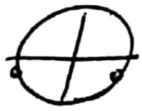
$$-\frac{\pi}{12},\ \frac{5\pi}{12},\ \frac{23\pi}{12}$$

۸-

$$-4\sin x\cos x = 1 \rightarrow -2\times\underbrace{2\sin x\cos x}_{\sin 2x} = 1$$

$$\sin 2x = -\frac{1}{2}$$

$$\sin(\frac{3\pi}{2}-x) = -\cos x$$



۲

$$\rightarrow 2x = 2k\pi+\frac{7\pi}{6} \rightarrow x = k\pi+\frac{7\pi}{12}$$

$$2x = 2k\pi+\pi-\frac{7\pi}{6}\;\left(-\frac{\pi}{6}\right) \rightarrow x = k\pi-\frac{\pi}{12}$$

$$\frac{11\pi}{12}\quad \frac{23\pi}{12}\quad \frac{7\pi}{12}\quad \frac{19\pi}{12}$$

$$5\pi$$

۹-

$$(1+\cancel{\cos 2x})(1+\cancel{\cos 4x})(1+\cancel{\cos 8x}) = \frac{1}{8}$$

$$2\cos^2 x \qquad 2\cos^2 2x \qquad 2\cos^2 4x$$

باتوجه $\cos^2\alpha = \dfrac{1+\cos 2\alpha}{2}$

$$\max \rightarrow \frac{8\pi}{9}$$

$$\xrightarrow{\times\sin^2\alpha} \underbrace{\underbrace{\underbrace{\sin^2\alpha\cos^2\alpha}_{\frac{1}{4}\sin^2 2\alpha}\cos^2 2\alpha}_{\frac{1}{16}\sin^2 4\alpha}\cos^2 4\alpha}_{\frac{1}{64}\sin^2 8\alpha}\times 8$$

$$\sin x\cos x = \frac{1}{2}\sin 2x$$

$$\sin^2\cos^2\alpha = \frac{1}{4}\sin^2 2\alpha$$

۲

$$\frac{1}{8}\,\frac{\sin^2 8\alpha}{\sin^2\alpha} = \frac{1}{8}$$

$$\sin^2 8\alpha = \sin^2\alpha$$

$$\sin 8\alpha = \pm\sin\alpha \rightarrow \sin 8\alpha = \sin\alpha$$

$$8\alpha = 2k\pi+\alpha \rightarrow 7\alpha = 2k\pi \rightarrow \alpha = \frac{2k\pi}{7}$$

$$8\alpha = 2k\pi+\pi-\alpha \rightarrow \alpha = \frac{2k\pi}{9}+\frac{\pi}{9}$$

$$\sin 8\alpha = -\sin\alpha$$

$$\sin 8\alpha = \sin(-\alpha)$$

$$8\alpha = 2k\pi-\alpha \rightarrow \alpha = \frac{2k\pi}{9}$$

$$8\alpha = 2k\pi+\pi+\alpha \rightarrow \alpha = \frac{2k\pi}{7}+\frac{\pi}{7}$$

$$\frac{\pi}{7},\ \frac{\pi}{9},\ \frac{4\pi}{7},\ \frac{6\pi}{7},\ \frac{3\pi}{9},\ \frac{5\pi}{9},\ \frac{7\pi}{9},\ \frac{4\pi}{9},\ \frac{6\pi}{9},\ \frac{2\pi}{9},\ \frac{8\pi}{9},\ \frac{2\pi}{7},\ \frac{5\pi}{7}$$

- ۱۰

۱,۷۵

$\tan(۳x)\tan(x) = ۱ \rightarrow \tan x = \frac{1}{\tan ۳x}$

$\tan ۳x = \frac{1}{\tan(x)}$

$\cot ۳x = \tan(\frac{\pi}{۲} - x)$

$\cancel{۳x = ۲k\pi + \pi}$

$۳x = k\pi + \frac{\pi}{۲} - x$

$۴x = k\pi + \frac{\pi}{۲} \rightarrow x = \frac{k\pi}{۴} + \frac{\pi}{۸}$ $[\pi, ۲\pi]$ بازه

$\frac{۹\pi}{۸}$, $\frac{۱۱\pi}{۸}$, $\frac{۱۳\pi}{۸}$, $\frac{۱۵\pi}{۸}$

$(\frac{۹+۱۱+۱۳+۱۵}{۸})\pi = \frac{۴۸\pi}{۸} = ۶\pi$