«تصحیح کل سرفصل» «مثلثات شماره ۱۲» «دوازدهم دختر A»

۱) $8\cos\alpha = \underbrace{\tan^2 x+1}_{\frac{1}{\cos^2\alpha}} \to \cos^3\alpha = \frac{1}{8} \to \cos\alpha = \frac{1}{2} \to \cos\alpha = \cos\frac{\pi}{3} \to \alpha = 2k\pi \pm \frac{\pi}{3}$

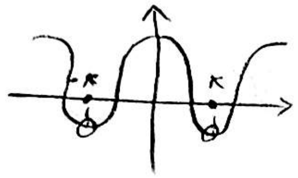
جواب ها: $\left\{\frac{\pi}{3}, \frac{5\pi}{3}\right\}$ ۲ جواب دارد

۲) $5\underbrace{\sin^2\alpha}_{1-\cos^2\alpha} + 2\cos\alpha = -2 \to 5 - 5\cos^2\alpha + 2\cos\alpha = -2 \to 5\cos^2\alpha - 2\cos\alpha - 7 = 0$

$\cos\alpha = -1$ ✓

$\cos\alpha = \frac{7}{5}$ غ ق ق

$\cos\alpha = -1 \to$



$\Rightarrow$ ۲ جواب دارد

۳) $(\underbrace{2\sin\alpha\cos2\alpha}) + \sin\alpha = 1$

$\sin\alpha\,(2\underbrace{\cos2\alpha}_{\cos^2-\sin^2}+1) = 1 \to 2\cos^2\sin - 2\sin^3 + \sin = \cancel{1} \to \cos^2+\sin^2$

$\sin(\underbrace{-2\sin^2+1}_{\cos2\alpha}) = \cos^2(1-2\sin\alpha) + \sin^2$

$= \frac{1-\cos2\alpha}{2}$

۴) $\frac{1}{\underset{\cos2x}{\sin(\frac{\pi}{2}+2x)}} + \frac{1}{\underset{-\sin4x}{\cos(\frac{\pi}{2}+4x)}} = \frac{-\sin4x+\cos2x}{-\sin4x\cos2x} = 0 \to +\sin4x = \cos2x$

$2\sin2x\,\cancel{\cos2x} = \cancel{\cos2x} \to \sin2x = \frac{1}{2} \to 2x = 2k\pi + \frac{\pi}{6} \to x = k\pi + \frac{\pi}{12}$

$\to 2x = 2k\pi + \frac{5\pi}{6} \to x = k\pi + \frac{5\pi}{12}$

جواب ها: $\left\{\frac{\pi}{12}, \frac{5\pi}{12}\right\}$

$\xrightarrow{\text{اختلاف}} \frac{5\pi}{12} - \frac{\pi}{12} = \frac{4\pi}{12} \to \boxed{\frac{\pi}{3}} \to \tan2\alpha \to \tan\frac{2\pi}{3} = \frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = -\sqrt{3}$

۵)

(۶) $\sin\left(x+\frac{\pi}{6}\right)\underbrace{\cos\left(x-\frac{\pi}{3}\right)}_{\frac{\pi}{3}-x \;\to\; \sin\left(x+\frac{\pi}{6}\right)}=1 \longrightarrow \sin^2\left(x+\frac{\pi}{6}\right)=1 \xrightarrow{1} \sin\left(x+\frac{\pi}{6}\right)=1 \to x+\frac{\pi}{6}=2k\pi+\frac{\pi}{2}$

$\longrightarrow x=2k\pi+\frac{\pi}{3}$

$\xrightarrow{2} \sin\left(x+\frac{\pi}{6}\right)=-1 \longrightarrow x+\frac{\pi}{6}=2k\pi+\frac{3\pi}{2}$

$\longrightarrow x=2k\pi+\frac{4\pi}{3}$

تعداد جواب‌ها: $\left\{\frac{\pi}{3}, \frac{4\pi}{3}\right\}$

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(۷) $\sin x+\sqrt{3}\cos x=\sqrt{2} \xrightarrow{\div 2} \underbrace{\frac{1}{2}}_{\sin\frac{\pi}{6}}\sin x+\underbrace{\frac{\sqrt{3}}{2}}_{\cos\frac{\pi}{6}}\cos x=\underbrace{\frac{\sqrt{2}}{2}}_{\cos\frac{\pi}{4}}$

$\longrightarrow \cos\left(x-\frac{\pi}{6}\right)=\cos\frac{\pi}{4} \longrightarrow x-\frac{\pi}{6}\overset{1}{=}2k\pi+\frac{\pi}{4} \to x=2k\pi+\frac{5\pi}{12}$

$\xrightarrow{2} x-\frac{\pi}{6}\overset{2}{=}2k\pi-\frac{\pi}{4} \to x=2k\pi-\frac{\pi}{12}$

تعداد جواب‌ها: $\left\{\frac{5\pi}{12}, -\frac{\pi}{12}, \frac{23\pi}{12}\right\} \xrightarrow{\text{جمع}} \underbrace{\frac{5\pi}{12}-\frac{\pi}{12}}_{\frac{4\pi}{12}}+\frac{23\pi}{12}=\frac{27\pi}{12}\to \frac{9\pi}{4}$

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(۸) $4\sin x\underbrace{\sin\left(\frac{3\pi}{2}-x\right)}_{-\cos x}=1 \longrightarrow -2\sin 2x=1 \longrightarrow \sin 2x=-\frac{1}{2} \xrightarrow{1} 2x=2k\pi-\frac{\pi}{6}$

$\longrightarrow x=k\pi-\frac{\pi}{12}$

$\xrightarrow{2} 2x=2k\pi-\frac{5\pi}{6} \to x=k\pi-\frac{5\pi}{12}$

جواب‌ها: $\left\{\frac{11\pi}{12}, \frac{23\pi}{12}, \frac{7\pi}{12}, \frac{19\pi}{12}\right\}$

جمع: $\frac{11\pi}{12}+\frac{23\pi}{12}+\frac{7\pi}{12}+\frac{19\pi}{12}=\frac{60\pi}{12}=5\pi$

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(۹) $\underbrace{(1+\cos 2\alpha)}_{\cancel{2}\cos^2\alpha}\underbrace{(1+\cos 4\alpha)}_{\times\cancel{2}\cos^2 2\alpha}\underbrace{(1+\cos 8\alpha)}_{\times\cancel{2}\cos^2 4\alpha}\times\frac{\sin^2\alpha}{\sin^2\alpha}=\frac{1}{8}\to\frac{1}{64}$

$\frac{1}{4}\sin^2 2\alpha\times\cos^2 2\alpha\times\cos^2 4\alpha \to \frac{1}{4}\times\frac{1}{4}\sin^2 4\alpha \to \cancel{\frac{1}{64}}\,\frac{\sin^2 8\alpha}{\sin^2\alpha}=\cancel{\frac{1}{64}}$

$\sin^2 8\alpha=\sin^2\alpha \xrightarrow{1} \sin 8\alpha=\sin\alpha \to 8\alpha=2k\pi+\alpha \to \alpha=\frac{2k\pi}{7}$

$\to 8\alpha=2k\pi+\pi-\alpha \to \alpha=\frac{2k\pi}{9}+\frac{\pi}{9}$

$\xrightarrow{2} \sin 8\alpha=-\sin\alpha$

$\to 8\alpha=2k\pi-\alpha \to \alpha=\frac{2k\pi}{9}$

$\to 8\alpha=2k\pi+\pi+\alpha \to \alpha=\frac{2k\pi}{7}+\frac{\pi}{7}$

$k=3$

$\boxed{\max: \pi}$

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(۱۰) $\tan 3x\tan x=1 \longrightarrow \tan 3x=\frac{1}{\tan x} \to \tan 3x=\underbrace{\cot x}_{\tan\left(\frac{\pi}{2}-x\right)} \to 3x=k\pi+\frac{\pi}{2}-x \to 4x=k\pi+\frac{\pi}{2}$

$\to x=\frac{k\pi}{4}+\frac{\pi}{8}$

جواب‌ها: $\left\{\frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8}\right\} \xrightarrow{\text{جمع}} \frac{48\pi}{8}=6\pi$