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تصحیح کل سرفصل «تکلیف شماره ۱۲» «دوازدهم دفتر A»

۱) $8\cos\alpha = \underbrace{\tan^2 x+1}_{\frac{1}{\cos^2\alpha}} \to \cos^3\alpha=\frac{1}{8} \to \cos\alpha=\frac{1}{2} \to \cos\alpha=\cos\frac{\pi}{3} \to \boxed{\alpha=2k\pi\pm\frac{\pi}{3}}$

جواب ها: $\{\frac{\pi}{3} , \frac{5\pi}{3}\}$ ۲ جواب داریم

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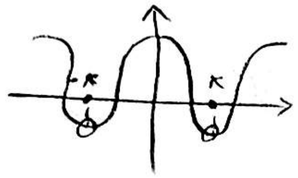
۲) $5\underbrace{\sin^2\alpha}_{1-\cos^2\alpha}+2\cos\alpha=-2 \to 5-5\cos^2\alpha+2\cos\alpha=-2 \to 5\cos^2\alpha-2\cos\alpha-7=0$

$\to \cos\alpha=-1$

$\to \cos\alpha=\frac{7}{5}$ غ ق ق

$\cos\alpha=-1 \to$



$\Rightarrow$ ۲ جواب داریم

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۳) $(\underbrace{2\sin\alpha\cos2\alpha})+\sin\alpha=1$

$\sin\alpha(2\underbrace{\cos2\alpha}_{\cos^2-\sin^2}+1)=1 \to 2\cos^2\sin-2\sin^3+\sin=1 \to \cos^2+\sin^2$

$\sin(\underbrace{-2\sin^2+1}_{\cos2\alpha}) = \cos^2(1-2\sin\alpha)+\sin^2$

$\frac{1-\cos2\alpha}{2}$

۴) $\frac{1}{\underbrace{\sin(\frac{\pi}{2}+2x)}_{\cos2x}}+\frac{1}{\underbrace{\cos(\frac{\pi}{2}+4x)}_{-\sin4x}} = \frac{-\sin4x+\cos2x}{-\sin4x\cos2x}=0 \to +\sin4x=\cos2x$

$2\sin2x\cancel{\cos2x}=\cancel{\cos2x} \to \sin2x=\frac{1}{2} \to 2x=2k\pi+\frac{\pi}{6} \to x=k\pi+\frac{\pi}{12}$

$\to 2x=2k\pi+\frac{5\pi}{6} \to x=k\pi+\frac{5\pi}{12}$

جواب ها: $\{\frac{\pi}{12} , \frac{5\pi}{12}\}$

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$\xrightarrow{\text{اختلاف}} \frac{5\pi}{12}-\frac{\pi}{12}=\frac{4\pi}{12} \to \boxed{\frac{\pi}{3}} \to \tan2\alpha \to \tan\frac{2\pi}{3}=\frac{\frac{\sqrt3}{2}}{\frac{-1}{2}}=-\sqrt3$

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۵) $\cos(x+\frac{\pi}{4})=\frac{\sqrt2}{2}(\cos x-\sin x)=\frac{\sqrt3}{3} \to \cos x-\sin x=\frac{\frac{1}{\sqrt3}}{\frac{1}{\sqrt2}}$

$\cos x-\sin x=\frac{\sqrt2}{\sqrt3} \xrightarrow{\text{گویا}} \frac{\sqrt6}{3} \to \boxed{\cos x-\sin x=\frac{\sqrt6}{3}}$

$(\cos x-\sin x)^2=1-\sin2x=\frac{6}{9} \to \boxed{\sin2x=\frac{1}{3}}$

$m(\cos x-\sin x)-3\sqrt6(\sin2x)=\sqrt6 \to \frac{\sqrt6}{3}m-3\sqrt6(\frac{1}{3})=\sqrt6$

$\frac{\sqrt6}{3}m-\sqrt6=\sqrt6 \to \frac{m\sqrt6}{3}=2\sqrt6 \to \boxed{m=6}$

۶) $\sin\left(x+\frac{\pi}{6}\right)\underbrace{\cos\left(x-\frac{\pi}{3}\right)}_{\frac{\pi}{3}-x} = 1 \longrightarrow \sin^2\left(x+\frac{\pi}{6}\right)=1 \xrightarrow{1} \sin\left(x+\frac{\pi}{6}\right)=1 \longrightarrow x+\frac{\pi}{6}=2k\pi+\frac{\pi}{2}$

$\sin\left(x+\frac{\pi}{6}\right)$

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$\longrightarrow x = 2k\pi + \frac{\pi}{3}$

$\xrightarrow{2} \sin\left(x+\frac{\pi}{6}\right) = -1 \longrightarrow x+\frac{\pi}{6} = 2k\pi+\frac{3\pi}{2}$

$\longrightarrow x = 2k\pi + \frac{4\pi}{3}$

تعداد جواب‌ها: $\left\{ \frac{\pi}{3}, \frac{4\pi}{3} \right\}$

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۷) $\sin x + \sqrt{3}\cos x = \sqrt{2} \xrightarrow{\div 2} \underbrace{\frac{1}{2}}_{\sin\frac{\pi}{6}}\sin x + \underbrace{\frac{\sqrt{3}}{2}}_{\cos\frac{\pi}{6}}\cos x = \underbrace{\frac{\sqrt{2}}{2}}_{\cos\frac{\pi}{4}}$

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$\longrightarrow \cos\left(x-\frac{\pi}{6}\right) = \cos\frac{\pi}{4} \longrightarrow x-\frac{\pi}{6} \overset{1}{=} 2k\pi + \frac{\pi}{4} \longrightarrow x = 2k\pi + \frac{5\pi}{12}$

$\overset{2}{\longrightarrow} x-\frac{\pi}{6} \overset{2}{=} 2k\pi - \frac{\pi}{4} \longrightarrow x = 2k\pi - \frac{\pi}{12}$

تعداد جواب‌ها: $\left\{ \frac{5\pi}{12}, -\frac{\pi}{12}, \frac{23\pi}{12} \right\} \xrightarrow{\text{جمع}} \underbrace{\cancel{\frac{5\pi}{12} - \frac{\pi}{12}}}_{\frac{4\pi}{12}} + \frac{23\pi}{12} = \frac{27\pi}{12} \rightarrow \frac{9\pi}{4}$

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۸) $4\sin x \underbrace{\cancel{\sin\left(\frac{3\pi}{2}-x\right)}}_{-\cos x} = 1 \longrightarrow -2\sin 2x = 1 \longrightarrow \sin 2x = -\frac{1}{2} \xrightarrow{1} 2x = 2k\pi - \frac{\pi}{6}$

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$\longrightarrow x = k\pi - \frac{\pi}{12}$

جواب‌ها: $\left\{ \frac{11\pi}{12}, \frac{23\pi}{12}, \frac{7\pi}{12}, \frac{19\pi}{12} \right\}$

$\xrightarrow{2} 2x = 2k\pi - \frac{5\pi}{6} \longrightarrow x = k\pi - \frac{5\pi}{12}$

جمع: $\frac{11\pi}{12} + \frac{23\pi}{12} + \frac{7\pi}{12} + \frac{19\pi}{12} = \frac{60\pi}{12} = 5\pi$

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۹) $\underbrace{(1+\cos 2\alpha)}_{2\cos^2\alpha}\underbrace{(1+\cos 4\alpha)}_{\times 2\cos^2 2\alpha}\underbrace{(1+\cos 8\alpha)}_{\times 2\cos^2 4\alpha} \times \frac{\sin^2\alpha}{\sin^2\alpha} = \frac{1}{8} \rightarrow \frac{1}{64}$

$\frac{\frac{1}{4}\sin^2 2\alpha \times \cos^2 2\alpha \times \cos^2 4\alpha}{\ldots}$, $\frac{1}{4}\times\frac{1}{4}\sin^2 4\alpha \rightarrow \frac{1}{64}\,\frac{\sin^2 8\alpha}{\sin^2\alpha} = \frac{1}{64}$

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$\sin^2 8\alpha = \sin^2\alpha \xrightarrow{1} \sin 8\alpha = \sin\alpha \longrightarrow 8\alpha = 2k\pi + \alpha \longrightarrow \alpha = \frac{2k\pi}{7}$

$\longrightarrow 8\alpha = 2k\pi + \pi - \alpha \longrightarrow \alpha = \frac{2k\pi}{9} + \frac{\pi}{9}$

$\xrightarrow{2} \sin 8\alpha = -\sin\alpha$

$\longrightarrow 8\alpha = 2k\pi - \alpha \longrightarrow \alpha = \frac{2k\pi}{9}$

$\longrightarrow 8\alpha = 2k\pi + \pi + \alpha \longrightarrow \alpha = \frac{2k\pi}{7} + \frac{\pi}{7}$

$k=3$

$\max: \pi$

* α نمی‌تواند برابر خود π باشد چون طبق * $\sin\alpha$ را مخالف صفر در نظر گرفته‌ایم!

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۱۰) $\tan 3x \tan x = 1 \longrightarrow \tan 3x = \frac{1}{\tan x} \longrightarrow \tan 3x = \underbrace{\cot x}_{\tan\left(\frac{\pi}{2}-x\right)} \longrightarrow 3x = k\pi + \frac{\pi}{2} - x \longrightarrow 4x = k\pi + \frac{\pi}{2}$

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$\longrightarrow x = \frac{k\pi}{4} + \frac{\pi}{8}$

جواب‌ها: $\left\{ \frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8} \right\} \xrightarrow{\text{جمع}} \frac{48\pi}{8} = 6\pi$

(۳)

$\cos 2x = 1 - 2\sin^2 x$ (فرمول طلایی)

۳ ٤ ۳

$2\sin x(1-2\sin^2 x) + \sin x = 1 \longrightarrow 3\sin x - 4\sin^3 x = 1$

$\sin 3x = 1 \longrightarrow 3x = 2k\pi + \frac{\pi}{2} \longrightarrow x = \frac{2k\pi}{3} + \frac{\pi}{6} \xrightarrow{0 \leq x \leq 2\pi}$

$k=0 \to x = \frac{\pi}{6}$, $k=1 \to x = \frac{5\pi}{6}$, $k=2 \to x = \frac{9\pi}{6}$

$S = \frac{\pi}{6} + \frac{5\pi}{6} + \frac{9\pi}{6} = \frac{15\pi}{6} = \boxed{\frac{5\pi}{2}}$

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$1 + \cos 2x = 2\cos^2 x$ ★

$\begin{cases} 1+\cos 2\alpha = 2\cos^2\alpha \\ 1+\cos 4\alpha = 2\cos^2 2\alpha \\ 1+\cos 8\alpha = 2\cos^2 4\alpha \end{cases} \longrightarrow 8\cos^2\alpha \cos^2 2\alpha \cos^2 4\alpha = \frac{1}{8}$

$\cos^2\alpha \cos^2 2\alpha \cos^2 4\alpha = \frac{1}{64} \xrightarrow{\sqrt{\ }} \cos\alpha \cos 2\alpha \cos 4\alpha = \frac{\pm 1}{8} \xrightarrow[\sin\alpha \neq 0]{\times \sin\alpha}$ ★

$\underbrace{(\sin\alpha \cos\alpha)}_{\frac{1}{2}\sin 2\alpha} \cos 2\alpha \cos 4\alpha = \pm \frac{\sin\alpha}{8} \longrightarrow \frac{1}{2}\underbrace{(\sin 2\alpha \cos 2\alpha)}_{\frac{1}{2}\sin 4\alpha} \cos 4\alpha = \pm\frac{\sin\alpha}{8}$

$\frac{1}{2} \times \frac{1}{2}\underbrace{(\sin 4\alpha \cos 4\alpha)}_{\sin 8\alpha} = \frac{\pm \sin\alpha}{8} \longrightarrow \frac{1}{\cancel{8}} \sin 8\alpha = \pm \frac{\sin\alpha}{\cancel{8}}$

$\sin 8\alpha = \pm \sin\alpha \longrightarrow \begin{cases} 8\alpha = 2k\pi + \alpha \ \text{یا} \ 2k\pi + \pi - \alpha \\ 8\alpha = 2k\pi - \alpha \ \text{یا} \ 2k\pi + \pi + \alpha \end{cases}$

$\alpha = \frac{2k\pi}{7} \xrightarrow{MAX\ A} k=3 \longrightarrow x = \frac{6\pi}{7}$

$\alpha = \frac{2k\pi}{9} \xrightarrow{MAX\ A} k=4 \longrightarrow x = \frac{8\pi}{9}$

$\alpha = \frac{(2k+1)\pi}{7} \xrightarrow{MAX\ A} k=2 \longrightarrow x = \frac{5\pi}{7}$

$\alpha = \frac{(2k+1)\pi}{9} \xrightarrow{MAX\ A} k=3 \longrightarrow x = \frac{7\pi}{9}$

$\longrightarrow \boxed{MAX\ A = \frac{8\pi}{9}}$

★ A نمی‌تواند برابر خود π باشد چون طبق ★ sinα را مخالف صفر در نظر گرفته‌ایم!