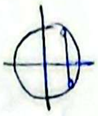
هانیه توانی روز دهم تست A

(۱) (۲)

$8\cos x-\tan^2x=1\rightarrow 1+\tan^2x=\frac{1}{\cos^2}$

$8\cos x=\frac{1}{\cos^2}\rightarrow 8\cos^3x=1$

$\cos x=\frac{1}{8}\rightarrow\cos x=\frac{1}{2}$



$x=2k\pi\pm\frac{\pi}{3}\rightarrow\frac{\pi}{3},\frac{5\pi}{3}$

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(۲)

$5\sin^2x+2\cos 3x=-2$

$5\sin^2x+2\cos 3x+2=0$

$5\sin^2x+2(\cos 3x+1)=0$

$\cos 3x+1=2\cos^2\frac{3x}{2}$

$\underbrace{5\sin^2x}_{0}+\underbrace{2\left(2\cos^2\frac{3x}{2}\right)}_{0}=0$

$5\sin^2x=0\rightarrow\sin x=0$ $\boxed{x=k\pi}\rightarrow 0,\pi,-\pi$

$4\cos^2\frac{3x}{2}=0\rightarrow\cos\frac{3x}{2}=0=\frac{\pi}{2}$

$\frac{3x}{2}=2k\pi+\frac{\pi}{2}\rightarrow\boxed{\frac{4k\pi}{3}+\frac{\pi}{3}}\rightarrow\frac{\pi}{3},-\pi$

$\frac{3x}{2}=2k\pi-\frac{\pi}{2}\rightarrow\boxed{\frac{4k\pi}{3}-\frac{\pi}{3}}\rightarrow-\frac{\pi}{3},\pi$

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(۳)

$2\sin x\cos 2x+\sin x=1$ | $\cos 2x=1-2\sin^2x$

$2\sin x(1-2\sin^2x)+\sin x=1$

$3\sin x-4\sin^3x=1\rightarrow\sin 3x=1$

$3x=2k\pi+\frac{\pi}{2}\rightarrow\frac{2k\pi}{3}+\frac{\pi}{6}$

$\boxed{\frac{\pi}{6},\frac{5\pi}{6},\frac{9\pi}{6}}\rightarrow\frac{15\pi}{6}$

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(۴)

$\frac{1}{\sin(\frac{\pi}{2}+2x)}+\frac{1}{\cos(\frac{\pi}{2}+4x)}=0$

$\cos(2x)$ $-\sin(4x)=\sin(-4x)$

$\sin(-4x)+\cos(2x)=0\rightarrow\cos(2x)=\sin(4x)$

$\cos(2x)=\cos(\frac{\pi}{2}-4x)$

$\overset{6x}{\cancel{2x}}=2k\pi+\frac{\pi}{2}-\cancel{4x}\rightarrow\frac{k\pi}{3}+\frac{\pi}{12}=\frac{4k\pi+\pi}{12}$

$\overset{-2x}{\cancel{2x}}=2k\pi-\frac{\pi}{2}+\cancel{4x}\rightarrow-k\pi+\frac{\pi}{4}$

$\downarrow\cancel{\frac{\pi}{4}}$ $\downarrow\frac{\pi}{12}$ $\frac{5\pi}{12}$ $\cancel{\frac{9\pi}{12}}$

$\frac{\pi}{4}\rightarrow\cos(\frac{\pi}{2}+\pi)=0$ ✗

$\frac{3\pi}{4}\rightarrow\sin(\frac{\pi}{2}+\frac{3\pi}{2})=0$ ✗

$\frac{9\pi}{12}-\frac{\pi}{12}=\frac{\pi}{3}$ $\tan(\frac{2\pi}{3})=-\sqrt{3}$

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(۵)

$m(\cos x-\sin x)-3\sqrt{6}\sin(2x)=\sqrt{6}$

$-m(\sqrt{2}\cancel{\sin(x-\frac{\pi}{4})})=\sqrt{6}(1+3\sin(2x))$

$\frac{1}{\sqrt{2}}$

$-m\sqrt{\frac{2}{18}}=1+3\sin(2x)$

$\frac{-m}{3}=1+3\sin(2x)$

$\rightarrow\sin(2x)=\frac{-3-m}{3}$

$\sin(x-\frac{\pi}{4})=\cos(x+\frac{\pi}{4})=\frac{1}{\sqrt{2}}$

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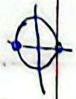
(۶) ۲

$\sin(x+\frac{\pi}{6})\cos(x-\frac{\pi}{4})=1$

$\cos^2(x-\frac{\pi}{4})=1\rightarrow\cos(x-\frac{\pi}{4})=\pm1$



$x-\frac{\pi}{4}=k\pi\rightarrow x=k\pi+\frac{\pi}{4}$ $x=\frac{\pi}{4}$ $\frac{5\pi}{4}$

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(۷)

$\sin x+\sqrt{3}\cos x=\sqrt{2}$ $\alpha\rightarrow\tan\alpha=\frac{\sqrt{3}}{3}$

$\frac{1}{|1|}\sqrt{1+3}\sin(x+\frac{\pi}{6})$

$2\sin(x+\frac{\pi}{6})=\frac{\sqrt{2}}{2}=\sin\frac{\pi}{4}$

$x=2k\pi+\frac{\pi}{4}-\frac{\pi}{6}=\frac{24k\pi+\pi}{12}$

$x=2k\pi+\frac{3\pi}{4}-\frac{\pi}{6}=\frac{24k\pi+7\pi}{12}$

$\frac{\pi}{12}$

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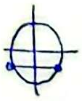
(۸)

$-4\sin x\cos x=1$

$2\sin x\cos x=\sin 2x=-\frac{1}{2}$



$2x=2k\pi-\frac{\pi}{6}\rightarrow x=k\pi-\frac{\pi}{12}$

$2x=2k\pi+\frac{5\pi}{6}\rightarrow x=k\pi+\frac{9\pi}{12}$

$x=\underbrace{\frac{11\pi}{12},\frac{23\pi}{12}}_{2\pi+\frac{5\pi}{6}},\underbrace{\frac{\pi}{2},\frac{3\pi}{2}}_{+2\pi}$ $3\pi+\frac{5\pi}{6}$

هانیه توکلی دوازدهم رشته A

⑨ $$(۱+\cos(۲\alpha))(۱+\cos(٤\alpha))(۱+\cos(۸\alpha)) = \frac{۱}{۸}$$

$$۲\cos^۲\alpha \qquad ۲\cos^۲۲\alpha \qquad ۲\cos^۲٤\alpha$$

$$\to \cos^۲\alpha \cdot \cos^۲۲\alpha \cdot \cos^۲۴\alpha = \frac{۱}{۶٤} \to (\underbrace{\cos\alpha\cos۲\alpha\cos۴\alpha}_{\frac{۱}{۸}})^۲ = \frac{۱}{۶٤}$$

$$\underbrace{\sin\alpha\cos\alpha}_{\frac{۱}{۲}\sin۲\alpha}\cos۲\alpha\cos٤\alpha = \frac{۱}{۸}\sin\alpha$$

$$\underbrace{\frac{۱}{۲}\sin۲\alpha \cdot \cos۲\alpha}_{\frac{۱}{٤}\sin۴\alpha}$$

$$\underbrace{\frac{۱}{٤}\sin۴\alpha \cdot \cos٤\alpha}_{\frac{۱}{۸}\sin۸\alpha}$$

$$\frac{۱}{۸}\sin۸\alpha \to \sin\alpha = \sin۸\alpha$$

$$\alpha = ۲k\pi + ۸\alpha$$

$$-\cancel{۷}\alpha = \frac{-۲k\pi}{۷} \xrightarrow[k=-۲]{} \frac{۴\pi}{۷}$$

$$\alpha = ۲k\pi - ۸\alpha$$

$$۹\alpha = \frac{۲k\pi}{۹} \xrightarrow[k=۴]{} \boxed{\frac{۸\pi}{۹}}$$ ✓

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⑩ $$\tan(۳x)\tan(x) = \frac{۱}{\tan(x)} \to \tan(۳x) = \cot(x) = \tan(\frac{\pi}{۲}-x)$$

$$۳x = k\pi + \frac{\pi}{۲} - x \to ۴x = k\pi - \frac{\pi}{۲} \to x = \frac{k\pi}{۴} - \frac{\pi}{۸}$$

$$\underbrace{\frac{۹\pi}{۸}، \frac{۱۱\pi}{۸}، \frac{۱۳\pi}{۸}، \frac{۱۵\pi}{۸}}_{\boxed{۴}}$$