مدرس ایرانی ۱

page: ( )

Subject:

Year: Month: Day:

۱- الف) $\cos 2x = 3\sin x - 1 \Rightarrow 1 - 2\sin^2 x = 3\sin x - 1 \Rightarrow 2\sin^2 x + 3\sin x - 2 = 0$

$\sin x = \dfrac{-3 \pm \sqrt{25}}{4} = \dfrac{-3 \pm 5}{4} = \underset{\text{غ ق ق}}{-2},\ \dfrac{1}{2} \Rightarrow \boxed{\begin{matrix} x = 2k\pi + \frac{\pi}{6} \\ x = 2k\pi + \frac{5\pi}{6} \end{matrix}}$

ب) $\cos 2x + \cos x - 2 = 0 \Rightarrow 2\cos^2 x + \cos x - 3 = 0 \Rightarrow \cos x = 1,\ \underset{\text{غ ق ق}}{-\frac{3}{2}}$

$\boxed{x = 2k\pi}$

۲- الف) $\sin^4 x - \cos^4 x = \sin 4x \Rightarrow \underbrace{(\sin^2 x + \cos^2 x)}_{1}\underbrace{(\sin^2 x - \cos^2 x)}_{-\cos 2x} = 2\sin 2x \cos 2x$

$\Rightarrow -1 = 2\sin 2x \Rightarrow \sin 2x = -\dfrac{1}{2} \Rightarrow 2x = 2k\pi + \dfrac{7\pi}{6} \Rightarrow \boxed{x = k\pi + \dfrac{7\pi}{12}}$

$2x = 2k\pi + \pi - \dfrac{7\pi}{6} = 2k\pi - \dfrac{\pi}{6} \Rightarrow \boxed{x = k\pi - \dfrac{\pi}{12}}$

ب) $2\cos^2 x + 2\sin x\cos x = 1 \Rightarrow \cos 2x + \sin 2x = 0 \Rightarrow \cos 2x = -\sin 2x = \sin(-2x)$

$= \cos\left(\dfrac{\pi}{2} - (-2x)\right)$

$\cos 2x = \cos\left(\dfrac{\pi}{2} + 2x\right) \Rightarrow 2x = 2k\pi + \dfrac{\pi}{2} + 2x$ غ ق ق

$2x = 2k\pi - \dfrac{\pi}{2} - 2x \Rightarrow 4x = 2k\pi - \dfrac{\pi}{2} \Rightarrow \boxed{x = \dfrac{k\pi}{2} - \dfrac{\pi}{8}}$

۳- الف) $\cot x(2\cos x + 1) = \dfrac{1}{\sin x} \Rightarrow \cos x(2\cos x + 1) = 1 \qquad , \sin \neq 0$

$\Rightarrow 2\cos^2 x + \cos x - 1 = 0 \Rightarrow \cos x = -1,\ \dfrac{1}{2} \Rightarrow x = 2k\pi + \pi$ غ ق ق $\Leftarrow \sin x = 0$

$\boxed{x = 2k\pi \pm \dfrac{\pi}{3}}$

ب) $2\sin^2 x - \sin x\cos x + \cos^2 x = 2 \Rightarrow \underbrace{2\sin^2 x - 2}_{\substack{2(\sin^2 x - 1) \\ -2\cos^2 x}} - \sin x\cos x + \cos^2 x = 0$

$\Rightarrow -\sin x\cos x - \cos^2 x = 0 \Rightarrow -\underset{\downarrow\ 0}{\cos x}\,\underset{\downarrow\ -1}{(\sin x + 1)} = 0$

$\left.\begin{matrix} x = k\pi + \frac{\pi}{2} \\ x = 2k\pi + \frac{3\pi}{2} \end{matrix}\right\} = \boxed{k\pi + \dfrac{\pi}{2}}$

۴

الف) $\cos(\frac{\pi}{4}+x)\cos(x-\frac{\pi}{4})=\frac{1}{4} \Rightarrow \cos(x+\frac{\pi}{4})\times\sin(x+\frac{\pi}{4})$

$=\frac{1}{2}\sin(2(x+\frac{\pi}{4}))=\frac{1}{2}\sin(2x+\frac{\pi}{2})=\frac{1}{4} \Rightarrow \sin(2x+\frac{\pi}{2})=\frac{1}{2} \Rightarrow \cos 2x=\frac{1}{2}$

$\Rightarrow 2x=2k\pi\pm\frac{\pi}{3} \Rightarrow x=\boxed{k\pi\pm\frac{\pi}{6}}$

ب) $\cos(3x-\frac{\pi}{9})=-\sin 2x=\sin(-2x)=\cos(\frac{\pi}{2}+2x)$

$\Rightarrow 3x-\frac{\pi}{9}=2k\pi+\frac{\pi}{2}+2x \Rightarrow$ غ ق ق

$3x-\frac{\pi}{9}=2k\pi-\frac{\pi}{2}-2x \Rightarrow 5x=2k\pi-\frac{\pi}{2}+\frac{\pi}{9} \Rightarrow \boxed{x=\frac{2k\pi}{5}-\frac{7\pi}{72}}$

۵

$\frac{\sin 4x+\sin 2x}{\sin 2x}-\sin^2 x=\cos^2 x \Rightarrow \frac{2\sin 2x\cos 2x+\sin 2x}{\sin 2x}=1$

$\Rightarrow \frac{\cancel{\sin 2x}(2\cos 2x+1)}{\cancel{\sin 2x}}=1 \Rightarrow 2\cos 2x+1=1 \Rightarrow 2\cos 2x=0 \quad 2x=k\pi+\frac{\pi}{2}$

$\boxed{x=\frac{k\pi}{2}+\frac{\pi}{4}}$

۶

$\frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}}=\frac{1}{\cancel{\frac{\sin\frac{x}{2}}{2}}}\ \cancel{+\cot\frac{x}{2}}$

$\frac{1+\cos\frac{x}{2}}{\sin\frac{x}{2}}$

$\Rightarrow \underbrace{\tan\frac{x}{4}=\frac{1}{\tan\frac{x}{4}}} \Rightarrow \tan^2\frac{x}{4}=1 \Rightarrow \tan\frac{x}{4}=\pm 1$

$\frac{x}{4}=\frac{k\pi}{2}+\frac{\pi}{4} \Rightarrow x=2k\pi+\pi$

$\sin\neq 0 \Rightarrow \frac{x}{2}\neq k\pi$ ✓ $\quad \Rightarrow x=\pi, -\pi$

$\cos\neq 0 \Rightarrow \frac{x}{2}\neq 2k\pi+\frac{\pi}{2}$ ✓ $\quad |\pi-(-\pi)|=\boxed{2\pi}$

۷

$\cos^2 x-\sin^2 x\cos^3 x=1 \Rightarrow \underbrace{\cancel{\cos^2 x-1}}_{-\sin^2 x}-\sin^2 x\cos^3 x=0$

$-\sin^2 x(1+\cos^3 x)=0 \Rightarrow \sin x=0 \Rightarrow x=0, \pi, 2\pi$

$1+\cos^3 x=0 \Rightarrow \cos^3 x=-1 \Rightarrow 3x=2k\pi+\pi \Rightarrow x=\frac{2k\pi}{3}+\frac{\pi}{3}$

۵ جواب $\quad \Rightarrow x=\frac{\pi}{3}, \pi, \frac{5\pi}{3}$

۸)

$$\frac{1-\tan x}{1+\tan x} = \tan 3x$$

$$\frac{\tan\frac{\pi}{4} - \tan x}{1 + \tan\frac{\pi}{4}\cdot\tan x} = \tan\left(\frac{\pi}{4} - x\right)$$

$$\Rightarrow \tan\left(\frac{\pi}{4} - x\right) = \tan 3x$$

$$\Rightarrow 3x = k\pi + \frac{\pi}{4} - x \Rightarrow x = \frac{k\pi}{4} + \frac{\pi}{16}$$

۹)

$$\sqrt{1+\sin 2x} = \sqrt{2}\cos 2x \Rightarrow \sqrt{1+2\sin x\cos x} = \sqrt{2}\,(\cos^2 x - \sin^2 x)$$

$$\Rightarrow |\cos x + \sin x| = \sqrt{2}\,(\cos^2 x - \sin^2 x) \Rightarrow \sqrt{2}\,|\cos x - \sin x| = 1$$

۱۰)

الف) $\sin^4 x - \cos^4 x = 1 \Rightarrow \sin^2 x - \cos^2 x = 1 \Rightarrow -\cos 2x = 1 \Rightarrow \cos 2x = -1$

$$\Rightarrow 2x = 2k\pi + \pi \Rightarrow x = k\pi + \frac{\pi}{2} \Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}$$

ب) $\sin^3 x - \cos^3 x = -1 \Rightarrow$ نقاط اصلی را چک می‌کنیم.

$x = 0 \Rightarrow 0 - 1 = -1$ ✓ $\quad x = \frac{\pi}{2} \Rightarrow 1 - 0 = 1 \quad x = \pi \Rightarrow 0 - (-1) = 1 \quad x = \frac{3\pi}{2} \Rightarrow -1 - 0 = -1$

$x = 2\pi \Rightarrow 0 - 1 = -1 \longrightarrow x = 2\pi, 0, \frac{3\pi}{2}$