فاطمه شعبانی

۱- الف)

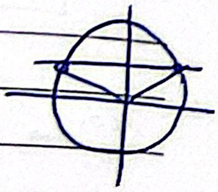
$\cos 2x = 3\sin x - 1$

$1 - 2\sin^2 x = 3\sin x - 1 \to 2\sin^2 x + 3\sin x - 2 = 0 \to \Delta = 25$

$\sin x \to -2$ ✗ $\to \sin x = \frac{1}{2} \to \begin{cases} x = 2k\pi + \frac{\pi}{6} \\ x = 2k\pi - \frac{\pi}{6} + \pi \end{cases}$

$\to \frac{1}{2}$ ✓

$x = 2k\pi + \frac{5\pi}{6}$



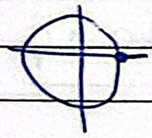
ب)

$\cos 2x + \cos x - 2 = 0$

$2\cos^2 x - 1 + \cos x - 2 = 0 \to 2\cos^2 x + \cos x - 3 = 0$

$\to \cos x \to \frac{c}{a} = -\frac{3}{2}$ ✗ $\to x = 2k\pi$

$\to 1$ ✓



۲- الف)

$\sin^4 x - \cos^4 x = \sin 4x$

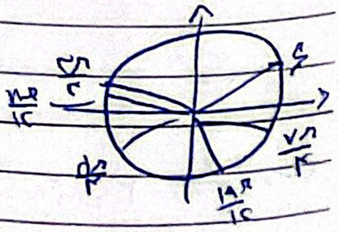
$\downarrow -\cos 2x \qquad \downarrow 2\sin 2x \cos 2x$

$\to 2\sin 2x \cos 2x + \cos 2x = 0 \to \cos 2x (1 + 2\sin 2x) = 0$

$\to \cos 2x = 0 \to \cos 2x = 0 \to x = k\pi + \frac{\pi}{4}$ ✓

$\to 2\sin 2x + 1 = 0 \to 2x \to 2k\pi - \frac{\pi}{6} \to x = k\pi - \frac{\pi}{12}$ ✓

$2x \to 2k\pi + \frac{5\pi}{6} \to x = \frac{3}{2}k\pi + \frac{\pi}{12}$ ✓



ب)

$2\cos^2 x + 2\sin x \cos x = 1$

$\underbrace{2\cos^2 x + 2\sin x\cos x}_{\cos 2x}{}^{\sin 2x} - 1 = 0 \rightarrow \cos 2x + \sin 2x = 0 \xrightarrow{(\;)^2} \underbrace{\cos^2 2x + \sin^2 2x}_{1} + \underbrace{2\sin 2x\cos 2x}_{\sin 4x}$

$= 0 \rightarrow \sin 4x = -1 \rightarrow x = \frac{k\pi}{2} + \frac{3\pi}{8}$

۳) الف)

$\cot x(2\cos x + 1) = \frac{1}{\sin x}, \quad \sin x \neq 0$

$\rightarrow \cos x(2\cos x + 1) = 1 \rightarrow 2\cos^2 x + \cos x - 1 = 0 \rightarrow \cos x = \frac{1}{2} \quad (1)$

$\cos x = -1 \quad (2)$

[illegible] ضرب کنم:

$1)\; x = 2k\pi \pm \frac{\pi}{3} \qquad 2)\; x = 2k\pi + \pi$

لغو امتیاز تنباکو به فتوای آیت‌الله میرزا حسن شیرازی (۱۲۷۰ هـ ش)

ب)

$2\sin^2 x - \sin x\cos x + \cos^2 x = 2$

یکی از این مشترکم

$1 + \sin^2 x - \sin x\cos x = 2 \rightarrow -1 + \sin^2 x + \sin x\cos x = 0 \xrightarrow{\div \cos^2 x}$ شرط $\cos x \neq 0$

$\rightarrow \tan^2 x + \tan x - \underbrace{1}_{\frac{1}{\cos^2 x}} - \tan^2 x = -\tan x - 1 = 0$

$\tan x = -1 \rightarrow x = k\pi - \frac{\pi}{4}$

شرط $\cos x = 0 \rightarrow 2\sin^2 x = 2 \rightarrow \sin^2 x = 1 \rightarrow \sin x = \pm 1 \rightarrow x = k\pi + \frac{\pi}{2}$

روز پاسداشت زبان فارسی و بزرگداشت حکیم ابوالقاسم فردوسی

شنبه ۲۷ اردیبهشت ۱۴۰۴ 17 May. 2025 ۱۹ ذی‌القعده ۱۴۴۶

م - الف)

$\cos(\frac{\pi}{4}+x)\cos(x-\frac{\pi}{4})=\frac{1}{4} \rightarrow$

$\cos(x+\frac{\pi}{4})\sin(x+\frac{\pi}{4})=\frac{1}{2}\sin(2x+\frac{\pi}{2})=\frac{1}{4} \rightarrow \cos 2x=\frac{1}{2}$

متمم‌اند

$\rightarrow \quad x=k\pi\pm\frac{\pi}{6}$

ب)

$\cos(2\pi-\frac{\pi}{9})=-\sin 2x$

$\cos(2x-\frac{\pi}{9})=\cos(\frac{\pi}{2}-2x) \rightarrow \cos(\frac{\pi}{2}-2x)+\cos(2x-\frac{\pi}{9})=0$

$\cos \alpha+\cos\beta=2\cos(\frac{\alpha+\beta}{2})\cos(\frac{\alpha-\beta}{2}) \rightarrow \cos(2x-\frac{\pi}{9})+\cos(\frac{\pi}{2}-2x)$

$=2\cos(\frac{7\pi}{36})\cos(2x-\frac{11\pi}{36})=0 \rightarrow 2x-\frac{11\pi}{36}=k\pi+\frac{\pi}{2}$

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$\rightarrow x=\frac{k\pi}{2}+\frac{29\pi}{72}$

۵-

$\frac{\sin 4x+\sin 2x}{\sin 2x}-\sin^2 x=\cos^2 x$

$\frac{\sin 4x}{\sin 2x}+1=\sin^2 x+\cos^2 x \rightarrow 2\cos 2x=0 \rightarrow 2\cos 2x=0$

$\cos 2x=0 \rightarrow x=k\pi\pm\frac{\pi}{4}$

۶-

$\frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}}=\frac{1}{\sin\frac{x}{2}}+\cot\frac{x}{2}$

$\tan\frac{x}{4}=\frac{1}{2}\tan\frac{x}{4}+\frac{1}{2}\cot\frac{x}{4}+\cot\frac{x}{2} \rightarrow t=\frac{1}{2}(t+\frac{1}{t})+\frac{1-t^2}{2t}$

$\rightarrow t=\frac{t^2+1}{2t}+\frac{1-t^2}{2t}=\frac{1}{t} \rightarrow t=\frac{1}{t} \rightarrow t^2=1 \rightarrow t=\pm 1$

$\rightarrow \tan\frac{x}{4}=\pm1 \rightarrow \frac{x}{4}=\frac{k\pi}{4}+\frac{\pi}{4} \rightarrow x=4k\pi+\pi$

روز بزرگداشت حکیم عمر خیام

حسین لطیعر

بازه $x \in [-\pi, \frac{3\pi}{2}]$ → $x = -\pi , \pi$ → $|\pi - (-\pi)| = 2\pi$

۷- $\cos^2 x - \sin^2 x \cos 4x = 1$

$-\sin^2 x \cos 4x = \underbrace{1 - \cos^2 x}_{\sin^2 x}$ → $\cos 4x = -1$ → $x = \frac{k\pi}{2} + \frac{\pi}{4}$

* $\sin^2 x \neq 0$ شرط

* $\sin^2 x = 0$ → $\underline{x = 2k\pi}$ → $x \in [0, 2\pi]$ → $\{0 , 2\pi\}$

۲ جواب

۸- $\frac{\tan x - 1}{\tan x + 1} = \tan 3x$ →

$\tan\left(\frac{\pi}{4} - x\right) = \frac{\tan\frac{\pi}{4} - \tan x}{1 + \tan\frac{\pi}{4}\tan x} = \frac{1 - \tan x}{1 + \tan x}$ → $\tan\left(\frac{\pi}{4} - x\right) = \tan 3x$

→ $\frac{\pi}{4} - x = 3x + k\pi$ → $\underline{x = \frac{\pi}{16} - \frac{k\pi}{4}}$

۹- $\sqrt{1 + \sin 2x} = \sqrt{2}\cos 2x$

$\sqrt{\underbrace{\sin^2 x + \cos^2 x + 2\sin x \cos x}_{(\sin x + \cos x)^2}} = \sqrt{2}\cos x$ → $|\sin x + \cos x| = \sqrt{2}\cos x$

→ $\sqrt{2}\left|\sin\left(x + \frac{\pi}{4}\right)\right| = \sqrt{2}\cos 2x$

شرط $\sin\left(x + \frac{\pi}{4}\right) = 0$: $\cos 2x = 0$ → $\underline{x = k\pi + \frac{\pi}{4}}$

شرط $\sin\left(x + \frac{\pi}{4}\right) \neq 0$: $2x = \frac{\pi}{4} - x + 2k\pi$

$3x = \frac{\pi}{4} + 2k\pi$ → $\underline{x = \frac{\pi}{12} + \frac{2k\pi}{3}}$

$x = \frac{3\pi}{2} \rightarrow -1 - 0 = -1$ ✓

$x = \frac{\pi}{2} \rightarrow 1 - 0 = 1$ ✗

$x = \pi \rightarrow 0 - (-1)$ ✗

۱۰- الف)

$\sin^4 x - \cos^4 x = 1 \rightarrow (\sin^2 x + \cos^2 x)(\sin^2 x - \cos^2 x) = 1$

$\quad 1 \qquad\qquad \cos^2 x = -1$

یا $\cos 2x = -1 \rightarrow \underline{x = k\pi + \frac{\pi}{2}} \xrightarrow{\in [0, 2\pi]} x = \left\{\frac{\pi}{2}, \frac{3\pi}{2}\right\}$

ب)

$\sin^3 x - \cos^3 x = -1$

با بررسی
نقاط کلیدی

$x = \rightarrow \frac{\pi}{2} \rightarrow 1 - 0 = 1$ ✗

$\underline{x = 0, \frac{3\pi}{2}}$

$0 \rightarrow 0 - 1 = \underline{-1}$ ✓ -1

$\frac{3\pi}{2} \rightarrow -1 - 0 = \underline{-1}$ ✓

$\pi \rightarrow 0 + 1 = 1$ ✗