

الف) $\cos 2x = 3 \sin x - 1$ $\xrightarrow{\cos 2x = 1 - 2 \sin^2 x}$ $1 - 2 \sin^2 x = 3 \sin x - 1 \Rightarrow 2 \sin^2 x + 3 \sin x - 2 = 0$ $\xrightarrow{a+b+c=0}$ $\sin^2 x + 3 \sin x - 2 = 0$

$$\Rightarrow \begin{cases} \sin x = \frac{1}{2} \\ \sin x = -2 \rightarrow \text{غیر ممکن} \end{cases} \Rightarrow \begin{cases} x = 2k\pi + \frac{\pi}{6} \\ x = 2k\pi + \frac{5\pi}{6} \end{cases}$$

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ب) $\cos 2x + \cos x - 2 = 0$ $\xrightarrow{\cos 2x = 2 \cos^2 x - 1}$ $2 \cos^2 x + \cos x - 3 = 0$ $\xrightarrow{a+b+c=0}$ $\begin{cases} \cos x = 1 \Rightarrow x = 2k\pi \\ \cos x = -\frac{3}{2} \rightarrow \text{غیر ممکن} \end{cases}$

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الف) $\sin^2 x - \cos^2 x = \sin x \cos x \Rightarrow \sin^2 x - \cos^2 x = 2 \sin x \cos x \xrightarrow{\sin^2 x - \cos^2 x = -\cos 2x}$ $-\cos 2x = 2 \sin x \cos x$

$$\Rightarrow 2 \sin x \cos x + \cos 2x = 0 \Rightarrow \cos 2x (2 \sin x + 1) = 0 \Rightarrow \begin{cases} \cos 2x = 0 \Rightarrow 2x = k\pi + \frac{\pi}{2} \Rightarrow x = \frac{k\pi}{2} + \frac{\pi}{4} \\ \sin x = -\frac{1}{2} \Rightarrow x = \begin{cases} 2k\pi - \frac{\pi}{6} \Rightarrow x = 2k\pi - \frac{\pi}{6} \\ 2k\pi - \frac{5\pi}{6} \Rightarrow x = 2k\pi - \frac{5\pi}{6} \end{cases} \end{cases}$$

ب) $2 \cos^2 x - 1 = -2 \sin x \cos x \xrightarrow{2 \cos^2 x - 1 = \cos 2x}$ $\cos 2x = -2 \sin x \cos x$ $\xrightarrow{-2 \sin x \cos x = -\sin 2x}$ $\cos 2x = \sin(-2x) = \cos(\frac{\pi}{2} + 2x)$

$$\begin{cases} 2x = 2k\pi + \frac{\pi}{2} + 2x \rightarrow x \\ 2x = 2k\pi - \frac{\pi}{2} - 2x \Rightarrow 4x = 2k\pi - \frac{\pi}{2} \Rightarrow x = \frac{k\pi}{2} - \frac{\pi}{4} \end{cases}$$

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الف) $\cos x (\cos x + 1) = \frac{1}{\sin x} \xrightarrow{\text{در صورتی که } \sin x \neq 0}$ $\cos x (\cos x + 1) = 1 \Rightarrow 2 \cos^2 x + \cos x - 1 = 0 \xrightarrow{a+b+c=0}$ $\begin{cases} \cos x = -1 \Rightarrow x = 2k\pi + \pi \\ \cos x = \frac{1}{2} \Rightarrow x = 2k\pi \pm \frac{\pi}{3} \end{cases}$

ب) $2 \sin^2 x + \cos^2 x - 1 = \sin x \cos x \Rightarrow \frac{\sin^2 x - 1}{-\cos^2 x} = \sin x \cos x \Rightarrow \cos^2 x + \sin x \cos x = 0 \Rightarrow \cos x (\cos x + \sin x) = 0$

$$\begin{cases} \cos x = 0 \Rightarrow x = k\pi + \frac{\pi}{2} \\ \cos x + \sin x = 0 \Rightarrow \cos x = -\sin x \Rightarrow \cos x = \cos(\frac{\pi}{2} + x) \Rightarrow \begin{cases} x = 2k\pi + \frac{\pi}{2} + x \rightarrow x \\ x = 2k\pi - \frac{\pi}{2} - x \Rightarrow 2x = 2k\pi - \frac{\pi}{2} \Rightarrow x = k\pi - \frac{\pi}{4} \end{cases} \end{cases}$$

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الف) $\cos(\frac{\pi}{2} + x) \cos(x - \frac{\pi}{2}) = \frac{1}{2} \rightarrow \cos(\frac{\pi}{2} + x) \cos(\frac{\pi}{2} - x) = \frac{1}{2}$ $\xrightarrow{\cos(\frac{\pi}{2} + x) = -\sin x}$ $-\sin x \cos(\frac{\pi}{2} - x) = \frac{1}{2}$ $\xrightarrow{\cos(\frac{\pi}{2} - x) = \sin x}$ $-\sin x \sin x = \frac{1}{2} \Rightarrow -\sin^2 x = \frac{1}{2} \Rightarrow \sin^2 x = -\frac{1}{2}$ $\Rightarrow \sin x = \pm \frac{1}{\sqrt{2}} \Rightarrow x = 2k\pi \pm \frac{\pi}{4}$

ب) $\cos(2x - \frac{\pi}{4}) = -\sin 2x = \sin(-2x) = \cos(\frac{\pi}{2} + 2x)$

$$\Rightarrow \begin{cases} 2x - \frac{\pi}{4} = 2k\pi + \frac{\pi}{2} + 2x \rightarrow x \\ 2x - \frac{\pi}{4} = 2k\pi - \frac{\pi}{2} - 2x \Rightarrow 4x = 2k\pi - \frac{\pi}{4} \Rightarrow x = \frac{k\pi}{2} - \frac{\pi}{8} \end{cases}$$

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$\frac{\sin^2 x + \sin^2 x}{\sin 2x} = \frac{\cos^2 x + \sin^2 x}{\sin 2x} \Rightarrow \frac{2 \sin^2 x}{\sin 2x} = 1 \Rightarrow 2 \cos^2 x + 1 = 1 \Rightarrow \cos^2 x = 0$

$$\Rightarrow 2x = k\pi + \frac{\pi}{2} \Rightarrow x = \frac{k\pi}{2} + \frac{\pi}{4}$$

ب) $\sin 2x \neq 0 \Rightarrow 2x \neq k\pi \Rightarrow x \neq \frac{k\pi}{2}$

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$$\frac{\sin \frac{x}{y}}{1 + \cos \frac{x}{y}} = \frac{1 + \cos \frac{x}{y}}{\sin \frac{x}{y}}$$

$$\tan \frac{x}{y} = \cot \frac{x}{y} \Rightarrow \cot \frac{x}{y} - \tan \frac{x}{y} = 0$$



$$\cot \frac{x}{y} = 0 \Rightarrow \frac{x}{y} = k\pi + \frac{\pi}{2} \Rightarrow x = yk\pi + \frac{y\pi}{2} \rightarrow \text{Unit circle}$$

در بازه $[-\pi, \pi]$

$$\rightarrow \{-\pi, \pi\}$$

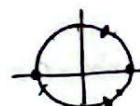
$$|-x - x| = \frac{y\pi}{2}$$

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$$\frac{-\sin^2 x \cos^2 x}{\sin^2 x} = \frac{1 - \cos^2 x}{\sin^2 x}$$

$$\Rightarrow \cos^2 x = -1 \Rightarrow x = yk\pi + \pi \Rightarrow x = \frac{yk\pi}{y} + \frac{\pi}{y}$$

$$\rightarrow \sin^2 x = 0 \Rightarrow \sin x = 0 \Rightarrow x = k\pi$$



$$\left\{ \pi, \frac{5\pi}{2}, \pi, \frac{\pi}{2}, 0 \right\} \leftarrow \text{جواب}$$

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$$1 = \tan \frac{\pi}{2} \quad \left| \frac{1 - \tan x}{1 + \tan x} = \frac{\tan \frac{\pi}{2} - \tan x}{1 + \tan \frac{\pi}{2} \tan x} = \tan \left(\frac{\pi}{2} - x \right) = \tan yx \Rightarrow yx = k\pi + \frac{\pi}{2} - x \right.$$

$$\Rightarrow x = k\pi + \frac{\pi}{2} \Rightarrow x = \frac{k\pi}{y} + \frac{\pi}{y}$$



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$$\sqrt{1 + \sin yx} = \sqrt{2} \cos yx \xrightarrow{\text{مربع کردن}} 1 + \sin yx = 2 \cos^2 yx \Rightarrow \sin yx = 2 \cos^2 yx - 1$$

$$\Rightarrow \cos yx = \cos \left(\frac{\pi}{2} - yx \right) \Rightarrow \begin{cases} yx = yk\pi + \frac{\pi}{2} - yx \Rightarrow yx = yk\pi + \frac{\pi}{2} \Rightarrow x = \frac{yk\pi}{y} + \frac{\pi}{2y} \\ yx = yk\pi - \frac{\pi}{2} + yx \Rightarrow yx = yk\pi - \frac{\pi}{2} \Rightarrow x = k\pi - \frac{\pi}{2y} \end{cases}$$



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بازای $x = \frac{5\pi}{4}$ $\cos x = \frac{\sqrt{2}}{2}$ $\sin x = \frac{\sqrt{2}}{2}$ $\cos 2x = \frac{\sqrt{2}}{2}$ $\sin 2x = \frac{\sqrt{2}}{2}$

جواب

$$\text{الف) } (\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x) = 1 \Rightarrow -\cos^2 x = 1 \Rightarrow \cos^2 x = -1 \Rightarrow yx = yk\pi + \pi$$

در صورت به فرم $\pm \sin^2 x \pm \cos^2 x$ داریم $-1 \leq 1$ فقط یکی است که با صفر

$$\sin^2 x - \cos^2 x = 1$$

$$x = k\pi$$

$$x = \frac{\pi}{2}$$



$$\Rightarrow x = k\pi + \frac{\pi}{2} \Rightarrow x = \frac{k\pi}{y} + \frac{\pi}{2y}$$



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$$\text{ب) } (\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x) = -1 \Rightarrow (\sin^2 x - \cos^2 x)(1 + \sin^2 x \cos^2 x) = -1$$