۱) [illegible] دوازدهم دسته A

۱) الف)

$\cos 2\alpha = 3\sin\alpha - 1$

$1 - 2\sin^2 x = 3\sin x - 1$

$2\sin^2 x + 3\sin x - 2 = 0$

$\sin x = \dfrac{-3 \pm \sqrt{\overset{5}{9+16}}}{4} = \begin{cases} \sin x = \frac{1}{2} \\ \sin x = \sin\frac{\pi}{6} \\ \sin x = 2 \times \end{cases}$

$\boxed{x = 2k\pi + \frac{\pi}{6}}$

$\boxed{x = 2k\pi + \frac{5\pi}{6}}$

ب)

$\cos 2x + \cos x - 2 = 0$

$2\cos^2 x - 1 + \cos x - 2 = 0$

$2\cos^2 x + \cos x - 3 = 0$

$\begin{cases} \cos x = 1 \\ \cos x = -\frac{3}{2} \times \end{cases}$ $\boxed{x = 2k\pi}$

۲) الف)

$\sin^4 x - \cos^4 x = \sin 4x$

$\underbrace{(\sin^2 x + \cos^2 x)}_{1}\underbrace{(\sin^2 x - \cos^2 x)}_{-\cos 2x} = \sin 4x$

$\cos(\pi - 2x) = \sin 4x = \cos(\frac{\pi}{2} - 4x)$

$\pi - 2x = 2k\pi + \frac{\pi}{2} - 4x$

$\boxed{x = k\pi - \frac{\pi}{4}}$

$\pi - 2x = 2k\pi - \frac{\pi}{2} + 4x$

$\boxed{x = -\frac{k\pi}{3} + \frac{\pi}{4}}$

ب)

$2\cos^2 x + 2\sin x\cos x = 1$

$2\sin x\cos x = 1 - 2\cos^2 x$

$\sin 2x = -\cos 2x = \cos(\pi - 2x)$

$\cos(\pi - 2x) = \cos(\frac{\pi}{2} - 2x)$

$\pi - 2x = 2k\pi - \frac{\pi}{2} + 2x$

$\boxed{x = -\frac{k\pi}{2} + \frac{3\pi}{8}}$

۳) الف)

$\cot x(2\cos x + 1) = \dfrac{1}{\sin x}$

$\dfrac{\cos x}{\cancel{\sin x}}(2\cos x + 1) = \dfrac{1}{\cancel{\sin x}}$ $\boxed{\sin x \neq 0}$

$2\cos^2 x + \cos x - 1 = 0$ $\begin{cases} \cos x = -1 \times \\ \cos x = \frac{1}{2} \end{cases}$

$\boxed{x = 2k\pi \pm \frac{\pi}{3}}$

ب)

$2\sin^2 x - \sin x\cos x + \cos^2 x = 2$

$\underbrace{\sin^2 x + \cos^2 x}_{1} + \sin^2 x - \sin x\cos x = 2\ (1)$

$\cancel{\sin^2 x} - \sin x\cos x = \cancel{\sin^2 x} + \cos^2 x$

$\cos^2 x + \sin x\cos x = 0$

$\cos x(\cos x + \sin x) = 0$

$\boxed{\sin x = \cos x}$

$\boxed{x = 2k\pi \pm \frac{\pi}{2}}$ $\boxed{x = k\pi + \frac{\pi}{4}}$

۴) الف)

$\cos(\frac{\pi}{4} + x)\cos(\frac{\pi}{4} - x) = \frac{1}{4}$

$\cos(\frac{\pi}{4} + x)\sin(\frac{\pi}{4} + x) = \frac{1}{4}$

$\cancel{\tfrac{1}{2}}\sin(\frac{\pi}{2} + 2x) = \cancel{\tfrac{1}{4}}\ \frac{1}{2}$

$\cos(2x) = \frac{1}{2}$

$2x = 2k\pi + \frac{\pi}{3}$

$\boxed{x = k\pi + \frac{\pi}{6}}$

$2x = 2k\pi - \frac{\pi}{3}$

$\boxed{x = k\pi - \frac{\pi}{6}}$

ب)

$\cos(\frac{\pi}{9} - 2x) = \sin(-2x)$

$\cos(\frac{\pi}{9} - 2x) = \cos(\frac{\pi}{2} + 2x)$

$\frac{\pi}{9} - 2x = 2k\pi + \frac{\pi}{2} + 2x$

$-4x = 2k\pi + \frac{7\pi}{18}$

$\boxed{x = -\frac{k\pi}{2} - \frac{7\pi}{72}}$

هانیه توئی　　　دوره دهم دفتر A

(5)

$$\frac{\sin 4x + \sin 2x}{\sin 2x} - \sin^2 x = \cos^2 x$$

$$\frac{\sin 4x + \sin 2x}{\sin 2x} = 1 \rightarrow \sin 4x + \sin 2x = \sin 2x$$

$$\sin 4x = 0 \quad \boxed{x = k\pi}$$

(6)

$$\frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}} = \frac{1}{\sin\frac{x}{2}} + \frac{\cos\frac{x}{2}}{\sin\frac{x}{2}}$$

$$\frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}} = \frac{1+\cos\frac{x}{2}}{\sin\frac{x}{2}}$$

$$\sin^2\frac{x}{2} = \left(1+\cos\frac{x}{2}\right)^2$$

$$\cancel{1} - \cos^2\frac{x}{2} = \cancel{1} + \cos^2\frac{x}{2} + 2\cos\frac{x}{2}$$

$$2\cos^2\frac{x}{2} + 2\cos\frac{x}{2} = 0$$

$$2\cos\frac{x}{2}\left(\cos\frac{x}{2}+1\right) = 0$$

$$\downarrow 0 \qquad \rightarrow -1$$

$$\frac{x}{2} = 2k\pi \pm \frac{\pi}{2} \qquad \frac{x}{2} = 2k\pi + \pi$$

$$\boxed{\begin{matrix} x = 4k\pi + \pi \\ x = 4k\pi - \pi \end{matrix}} \qquad \times\ \boxed{x = 4k\pi + 2\pi}$$

$$x = \pi \quad x = -\pi \qquad \boxed{|\pi - (-\pi)| = 2\pi}$$

(7) ت 5

$$\cos^2 x - \sin^2 x \cos 4x = 1$$

$$\cancel{1} - \sin^2 x - \sin^2 x \cos 4x = \cancel{1}$$

$$-\sin^2 x(1 + \cos 4x) = 0$$

$$\downarrow 0 \qquad \rightarrow \cos 4x = -1$$

$$\boxed{x = k\pi} \qquad 0, \pi, 2\pi$$

$$4x = 2k\pi + \pi$$

$$\boxed{x = \frac{2k\pi}{4} + \frac{\pi}{4}}$$

$$\frac{\pi}{4}, \pi, \frac{5\pi}{4}$$

(8)

$$\frac{1-\tan x}{1+\tan x} = \tan 3x$$

$$\frac{\frac{\cos x - \sin x}{\cancel{\cos x}}}{\frac{\cos x + \sin x}{\cancel{\cos x}}} = \frac{\cancel{\sqrt{2}}\sin\left(\frac{\pi}{4} - x\right)}{\cancel{\sqrt{2}}\sin\left(\frac{\pi}{4} + x\right)} = \tan 3x$$

$$\frac{\sin\left(\frac{\pi}{4} - x\right)}{\cos\left(\frac{\pi}{4} - x\right)} = \tan\left(\frac{\pi}{4} - x\right) = \tan 3x$$

$$\frac{\pi}{4} - x = k\pi + 3x \rightarrow -4x = k\pi - \frac{\pi}{4}$$

$$\boxed{x = -\frac{k\pi}{4} + \frac{\pi}{16}}$$

(9) ت 3

$$\sqrt{1 + \sin 2x} = \sqrt{2}\cos 2x$$

$$1 + \sin 2x = 2(\cos^2 2x) \quad (1 - \sin^2 2x)$$

$$1 + \sin 2x = 2 - 2\sin^2 2x$$

$$2\sin^2 2x + \sin 2x - 1 = 0$$

$$\begin{cases} \sin 2x = -1 & = 2k\pi - \frac{\pi}{2} \\ \sin 2x = \frac{1}{2} & = 2k\pi + \frac{\pi}{6} \end{cases} \qquad 2k\pi + \frac{5\pi}{6}$$

$$x = k\pi - \frac{\pi}{4} \qquad x = k\pi + \frac{\pi}{12} \qquad x = k\pi + \frac{5\pi}{12}$$

$$\frac{3\pi}{4} \qquad \frac{\pi}{12} \qquad \frac{5\pi}{12}$$

(10)

ب)

$$\sin^3 x - \cos^3 x = -1$$

$$(\sin x - \cos x)\left(\sin^2 x + \cos^2 x + \frac{1}{2}\sin 2x\right) = -1$$

$$\downarrow$$

$$\sqrt{2}\sin\left(\frac{\pi}{4} - x\right)$$

ت1)

$$\sin^4 x - \cos^4 x = 1$$

$$(\cancel{\sin^2 x + \cos^2 x})(\sin^2 x - \cos^2 x) = 1$$

$$\cos 2x = -1$$

$$2x = 2k\pi + \pi$$

$$\boxed{x = k\pi + \frac{\pi}{2}}$$

$$\frac{\pi}{2} \qquad \frac{3\pi}{2}$$