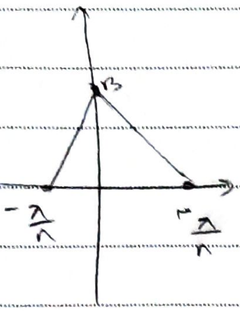
۱-



$y = \tan(-2x + \frac{\pi}{4})$

$B: x=0 \rightarrow \tan\frac{\pi}{4} = 1 \rightarrow B(0,1)$

$A, C: \tan(-2x + \frac{\pi}{4})$ تن

$\rightarrow \tan(-2x+\frac{\pi}{4}) = \tan\frac{\pi}{2} \rightarrow x = -\frac{\pi}{8}$

$\tan(-2x+\frac{\pi}{4}) = \tan(-\frac{\pi}{2}) \rightarrow x = \frac{3\pi}{8}$

$\dfrac{1\times(\frac{3\pi}{8} - (-\frac{\pi}{8}))}{2} = \boxed{\frac{\pi}{4}}$

۲-

$x^2 - (\sin\alpha)x - \frac{1}{4}\cos^2\alpha = 0 \xrightarrow{x=\frac{2}{3}} \frac{4}{9} - \frac{2}{3}\sin\alpha - \frac{1}{4}(1-\sin^2\alpha) = 0$

$= 9\sin^2\alpha - 24\sin\alpha + 7 = 0 \qquad \sin\alpha = \dfrac{24 \pm \sqrt{576-252}}{18} = \dfrac{24 \pm 18}{18} \Rightarrow \frac{42}{18}$ غ ق ق

$\frac{6}{18} = \frac{1}{3}$ ق ق

$\cos^2\alpha = 1 - \sin^2 = 1 - \frac{1}{9} = \frac{8}{9}$

$1 + \tan^2\alpha = \dfrac{1}{\cos^2\alpha} = \boxed{\frac{9}{8}}$

۳-

$\log^{\sin\alpha} - \log^{-\cos\alpha} = \log^{3} \rightarrow \log^{\frac{\sin\alpha}{-\cos\alpha}} = \log^{3} \Rightarrow \sin\alpha = -3\cos\alpha$

$\sin^2\alpha + \cos^2\alpha = 1 \rightarrow 9\cos^2\alpha + \cos^2\alpha = 1 \rightarrow \cos^2\alpha = \frac{1}{10} \rightarrow \cos\alpha = -\frac{\sqrt{10}}{10} \qquad \sin\alpha = \frac{3\sqrt{10}}{10}$

$\sin\alpha - \cos\alpha = \frac{3\sqrt{10}}{10} - (-\frac{\sqrt{10}}{10}) = \frac{4\sqrt{10}}{10} = \boxed{\frac{2\sqrt{10}}{5}}$

۴-

$\sin^4\theta + \cos^2\theta = \frac{4}{5} \rightarrow \sin^4\theta + 1 - \sin^2\theta = \frac{4}{5} \rightarrow \sin^4\theta - \sin^2\theta = -\frac{1}{5}$

$\sin^2\theta(\sin^2\theta - 1) = \frac{1}{5} \rightarrow \sin^2\theta\cos^2\theta = \frac{1}{5}$

$\cos^4\alpha + \sin^2\alpha = \cos^4\theta + 1 - \cos^2\alpha = \cos^2(\cos^2 - 1) + 1 = -\sin^2\theta\cos^2\theta + 1 = -\frac{1}{5} + 1 = \boxed{\frac{4}{5}}$

۵-

$\dfrac{3\sin x + \sin x\cos x}{2 - 2\cos^2 x} < 0 \rightarrow \dfrac{\sin x(3+\cos x)}{2(\underbrace{1-\cos^2 x}_{\sin^2 x})} = \dfrac{\overbrace{3+\cos x}^{+}}{2\sin x} < 0$

$\rightarrow 2\sin x < 0 \rightarrow \sin x < 0 \Rightarrow$

$\sin x\tan x - \dfrac{1}{\cos x} > 0 \rightarrow \dfrac{1}{\cos x}(\underbrace{\sin^2 x - 1}_{-\cos^2 x}) > 0 \rightarrow -\cos x > 0 \rightarrow \cos x < 0 \Rightarrow$

$\Rightarrow$ $\boxed{\text{ناحیه ۳}}$

۶ـ $\frac{2}{\sin x} + \frac{3}{\cos x} = 0 \Rightarrow \frac{2\cos x + 3\sin x}{\sin x \cos x} = 0 \rightarrow 2\cos x + 3\sin x = 0$

$\Rightarrow \cos x = -\frac{3}{2}\sin x \qquad \sin^2 x + \cos^2 x = 1 \Rightarrow \sin^2 x + \frac{9}{4}\sin^2 x = 1$

$\Rightarrow \sin^2 x = \frac{4}{13} \Rightarrow \sin x = \pm \frac{2}{\sqrt{13}} = \pm \frac{2\sqrt{13}}{13} \Rightarrow \cos x = \mp \frac{3\sqrt{13}}{13}$ مختلف العلامه

$\tan x + \cot x = \frac{\sin^2 + \cos^2}{\sin\cos} = \frac{1}{\sin\cos} = \frac{1}{-\frac{2\sqrt{13}}{13} \times \frac{3\sqrt{13}}{13}} = \boxed{-\frac{13}{6}}$

۷ـ $1 + \sin 40 = 1 + 2\sin 20 \cos 20 = (\sin 20 + \cos 20)^2 \Rightarrow \sqrt{1 + \sin 40} = \sin 20 + \cos 20$

$= \sqrt{2}\sin(20 + 45) = \sqrt{2}\sin 65 = \sqrt{2}\cos 25$

$\sin 50 + \sin 10 = \sin(20 + 30) + \sin(30 - 20) = \cancel{\sin 20 \cos 30} + \sin 30 \cos 20 + \sin 30 \cos 20$

$\cancel{- \sin 20 \cos 30} = 2\underbrace{\sin 30}_{\frac{1}{2}} \cos 20 = \cos 20 \qquad \boxed{\frac{\sqrt{2}\cos 25}{\cos 25} = \sqrt{2}}$

۸ـ $\frac{1 - \tan^2 20}{1 + \tan^2 20} = \cos 40 \qquad \cos 10 \times \cos 20 \times \cos 40 \xrightarrow[\div \sin 10]{\times \sin 10}$

$= \frac{1}{2}\sin 20 = \frac{1}{4}\sin 40 = \frac{1}{8}\frac{\sin 80}{\sin 10} = \frac{1}{8}\frac{\sin(\frac{\pi}{2} - 10)}{\sin 10} = \boxed{\frac{1}{8}\cot 10}$

۹ـ $\cos 2\alpha = 2\cos^2\alpha - 1 \Rightarrow 5\cos 2\alpha - 12\cos\alpha = 10\cos^2\alpha - 12\cos\alpha - 5$

$\sin\alpha + 3\cos\alpha = 2 \Rightarrow \sin\alpha = 2 - 3\cos\alpha \Rightarrow \sin^2\alpha + \cos^2\alpha = 1$

$(2 - 3\cos\alpha)^2 + \cos^2\alpha = 9\cos^2\alpha + 4 - 12\cos\alpha + \cos^2\alpha = 10\cos^2\alpha - 12\cos\alpha + 4 = 1$

$10\cos^2\alpha - 12\cos\alpha = -3 \Rightarrow 10\cos^2\alpha - 12\cos\alpha - 5 = -3 - 5 = \boxed{-8}$

۱۰ـ $\cos^2\frac{A}{2} = \frac{1 + \cos A}{2} \qquad \cos A = \cos(\pi - (B + C)) = -\cos(B + C)$

$\Rightarrow 1 - \cos(B + C) = 2\sin B \sin C \qquad 1 - (\cos B \cos C - \sin B \sin C) = 2\sin B \sin C$

$1 - \cos B \cos C + \sin B \sin C = 2\sin B \sin C$

$1 - (\cos B \cos C + \sin B \sin C) = 0 \Rightarrow 1 - \cos(B - C) = 0 \Rightarrow \cos(B - C) = 1$

$\Rightarrow B - C = 0 \Rightarrow B = C = 72 \qquad A = 180 - (2 \times 72) = \boxed{36}$