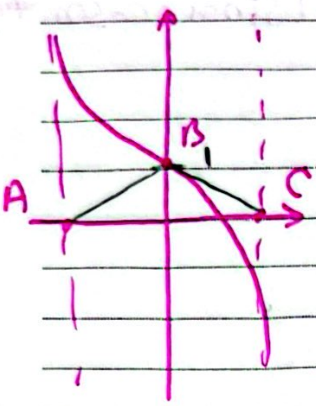
فاطمه شعبانی

۱) $y = \tan(-2x + \frac{\pi}{4}) \to$ مساحت $ABC = ?$



$\hookrightarrow \tan(-2(0)+\frac{\pi}{4}) = \tan(\frac{\pi}{4}) = 1 \to B = (0,1)$

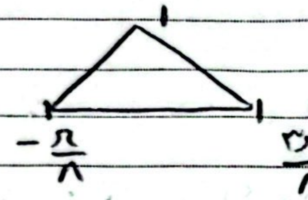
۲ انتهای تابع را که متقارن نسبت به y باشند بدست می‌آوریم:

$\tan(-2x + \frac{\pi}{4}) = 0 \Rightarrow$

$+\ -2x+\frac{\pi}{4} = \frac{\pi}{2} \to -2x = \frac{\pi}{4} \to x = -\frac{\pi}{8}$

$+\ -2x+\frac{\pi}{4} = \frac{-\pi}{2} \to -2x = \frac{-3\pi}{4}$

$\to -2x = -\frac{3\pi}{4} \to x = \frac{3\pi}{8}$



$\to \frac{\frac{4\pi}{8}\times 1}{2} = \boxed{\frac{\pi}{4} = S}$

۲) جواب‌های $x^2 - (\sin\alpha)x - \frac{1}{4}\cos^2\alpha = 0$ ریشه $\frac{2}{3} \to 1+\tan^2\alpha$

$x^2 - (\sin\alpha)x - \frac{1}{4}\cos^2\alpha = 0$

$\hookrightarrow \frac{4}{9} - \frac{2}{3}\sin\alpha - \frac{1}{4}\cos^2\alpha = 0 \xrightarrow{\times 36} 16 - 24\sin\alpha \underbrace{-9\cos^2\alpha}_{-9(1-\sin^2\alpha)} = 0$

$\to 9\sin^2\alpha - 24\sin\alpha + 7 = 0 \xrightarrow{\Delta} \sin\alpha \to \frac{24+18}{18} > 1 \to ✗$

$\hookrightarrow \sin\alpha = \frac{24-18}{18} = \frac{1}{3} ✓$

$\to 1+\tan^2\alpha = \frac{1}{\cos^2\alpha} = \frac{1}{\frac{8}{9}} = \boxed{\frac{9}{8}}$

* $\sin\alpha = \frac{1}{3}$

$\bigcirc \to \sqrt{8}$

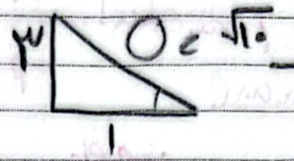
$\to \cos\alpha = \frac{\sqrt{8}}{3}$

$\sin\alpha > 0$ ، $\cos\alpha < 0$

۳) $\log(\sin\alpha) - \log(-\cos\alpha) = \log 3$ ، $\sin\alpha - \cos\alpha$ چیست؟

$\hookrightarrow \log\left(\frac{\sin\alpha}{-\cos\alpha}\right) = \log(-\tan\alpha) = \log 3 \rightarrow$

$\tan\alpha = -3 \rightarrow$



$\rightarrow \sin\alpha = \frac{3}{\sqrt{10}}$ ، $\cos\alpha = -\frac{1}{\sqrt{10}}$

([illegible])

$\rightarrow \sin\alpha - \cos\alpha = \frac{3}{\sqrt{10}} + \frac{1}{\sqrt{10}} = \frac{4}{\sqrt{10}} = \frac{4\sqrt{10}}{10} = \boxed{\frac{2\sqrt{10}}{5}}$

۴) $\sin^4\theta + \cos^2\theta = \frac{4}{5} \rightarrow \cos^4\theta + \sin^2\theta$ ؟

$\hookrightarrow \sin^4\theta + 1 - \sin^2\theta = \frac{4}{5} \rightarrow \underline{\sin^4\theta - \sin^2\theta + 1 = \frac{4}{5}}$

$\rightarrow \underbrace{\cos^4\theta}_{(1-\sin^2\theta)^2} + \sin^2\theta = \sin^4\theta - 2\sin^2\theta + 1 + \sin^2\theta =$

$\underline{\sin^4\theta - \sin^2\theta + 1} = \boxed{\frac{4}{5}}$

۵) $\frac{3\sin x + \sin x\cos x}{2 - 2\cos^2 x} < 0$ و $\sin x\tan x - \frac{1}{\cos x} > 0 \rightarrow$ ناحیه؟

($\sin x$ و $\cos x \neq 0$)

$\rightarrow \frac{\cancel{\sin x}(\cos x + 3)}{2(\underbrace{1-\cos^2 x}_{\sin^2 x})} = \frac{\overbrace{\cos x + 3}^{2 \le \;\; \le 4 \to \oplus}}{2\sin x} < 0 \rightarrow \underline{\sin x < 0}$ I ربع ۳ و ۴

$\sin x \underbrace{\tan x}_{\frac{\sin x}{\cos x}} - \frac{1}{\cos x} > 0 \rightarrow \frac{\sin^2 x - 1}{\cos x} > 0 \rightarrow \frac{-\cos^2 x}{\cos x} > 0 \rightarrow$

$-\cos x > 0 \rightarrow$

II $\cos x < 0 \rightarrow$ ۲ و ۳

$\mathrm{I} \cap \mathrm{II} \rightarrow \boxed{\text{ناحیه ۳}}$

(6

$$\frac{2}{\sin x}+\frac{3}{\cos x}=0 \rightarrow \tan x+\cot x=?$$

$(\sin x, \cos x \neq 0)$

$$\frac{2\cos x+3\sin x}{\sin x\cos x}=0 \rightarrow 2\cos x+3\sin x=0 \rightarrow$$

$$2\cos x=-3\sin x \rightarrow \frac{\sin x}{\cos x}=-\frac{2}{3} \rightarrow$$

$$\tan x+\cot x= -\frac{2}{3}-\frac{3}{2}=\boxed{\frac{-13}{6}}$$

(7

$$\frac{\sqrt{1+\sin 50^\circ}}{\sin 50^\circ+\sin 10^\circ}=?$$

$$* \ 1+\sin\theta=\left(\cos\frac{\theta}{2}+\sin\frac{\theta}{2}\right)^2$$

$$1+2\times\frac{1}{2}\sin\theta=1+\sin\theta$$

$$\frac{\sin 25^\circ+\cos 25^\circ}{\sin 50^\circ+\sin 10^\circ}$$

$$\rightarrow =\sqrt{2}\sin(45+25)=\sqrt{2}\sin 70^\circ=\sqrt{2}\cos 20^\circ$$

$$=2\sin 30^\circ\cos 20^\circ=\cos 20^\circ$$

$$\rightarrow \frac{\sqrt{2}\cos 20^\circ}{\cos 20^\circ}=\boxed{\sqrt{2}}$$

$$* \ \sin\alpha+\sin\beta=2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

(8

$$(1-2\sin^2 5^\circ)(2\cos^2 10^\circ-1)\left(\frac{1-\tan^2 20^\circ}{1+\tan^2 20^\circ}\right)=?$$

$$1-2\sin^2 5^\circ \rightarrow 1-(1-\cos 10^\circ) \qquad 2\cos^2 10^\circ-1 \rightarrow \cos 20^\circ \qquad \rightarrow \cos 40^\circ$$

$$=\cos 40^\circ\times\cos 20^\circ\times\cos 10^\circ$$

$$\sin^2\theta=\frac{1-\cos 2\theta}{2} \qquad \cos^2 10^\circ=\frac{1+\cos 20^\circ}{2}$$

$$\times\frac{\sin 10^\circ}{\sin 10^\circ} \rightarrow \frac{\overset{\frac{1}{8}\sin 80^\circ}{\overset{\frac{1}{4}\sin 40^\circ}{\overset{\frac{1}{2}\sin 20^\circ}{\sin 10^\circ\cos 10^\circ}}\times\cos 20^\circ\times\cos 40^\circ}}{\sin 10^\circ}=\frac{\frac{1}{8}\cos 10^\circ}{\sin 10^\circ}=\boxed{\frac{\cot 10^\circ}{8}}$$

$\sin\alpha + ۳\cos\alpha = ۲ \to ۵\cos ۲\alpha - ۱۲\cos\alpha = ?$ (۹

$\sin\alpha = ۲ - ۳\cos\alpha \quad \xrightarrow{(\;)^۲} \quad \sin^۲\alpha + ۹\cos^۲\alpha + ۶\cos\alpha\sin\alpha = ۴$

$\to ۱ - \cos^۲\alpha + ۹\cos^۲\alpha + ۶\cos\alpha(۲ - ۳\cos\alpha) = ۴$

مرتب

$\to -۱۰\cos^۲\alpha + ۱۲\cos\alpha = ۳$ **

$\to ۵\underbrace{\cos ۲\alpha}_{\cos^۲\alpha = \frac{۱+\cos ۲\alpha}{۲}} - ۱۲\cos\alpha = \underbrace{۵(۲\cos^۲\alpha - ۱) - ۱۲\cos\alpha}_{۱۰\cos^۲\alpha - ۵ - ۱۲\cos\alpha} = \boxed{-۸}$

$۱۰\cos^۲\alpha - ۵ - ۱۲\cos\alpha$

** $-۳ - ۵$

$\sin B \times \sin C = \cos^۲\frac{A}{۲} \to \sin B + \sin C = \frac{۱+\cos A}{۲}$ (۱۰

$\to \sin B \times \sin C = \frac{۱+\cos A}{۲} \to ۲\sin B \times \sin C = ۱ + \cos A$

$۲\left(\frac{۱}{۲}\left(\underbrace{\cos(B-C) - \cos(\pi - A)}\right)\right) = \cos A + ۱$

$\cos(B-C) + \cos A = \cos A + ۱ \to \cos(\underbrace{B-C}_{=۰}) = ۱$

$\to B = C \to ۱۸۰ - ۱۴۴ = \boxed{A = ۳۶^\circ}$