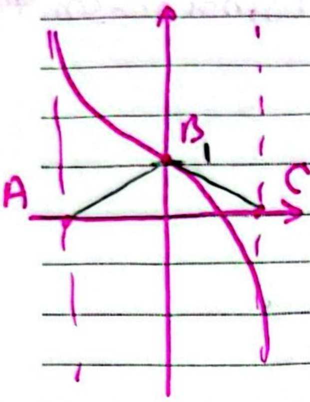
فاطمه شعبانی

۱) [illegible] ۲۰ — $y=\tan(-2x+\frac{\pi}{4}) \to$ مساحت $\hat{ABC} = ?$



$\to \tan(-2(0)+\frac{\pi}{4}) = \tan(\frac{\pi}{4}) = 1 \to B=(0,1)$

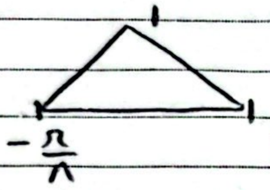
۲ انتهای تابع را که متقارن [illegible] می‌کنیم:

$\tan(-2x+\frac{\pi}{4}) = 0 \Rightarrow \to$

$+\ -2x+\frac{\pi}{4} = \frac{\pi}{2} \to -2x = \frac{\pi}{4} \to x = \frac{-\pi}{8}$

$+\ -2x+\frac{\pi}{4} = \frac{-\pi}{2} \to -2x = \frac{-3\pi}{4}$

۲ ✓ $\to -2x = \frac{-3\pi}{4} \to x = \frac{3\pi}{8}$



$\to \frac{\frac{4\pi}{8}\times 1}{2} = \boxed{\frac{\pi}{4} = S}$

۲) حاصل $1+\tan^2\alpha$ ← $\frac{2}{3}$ ریشه‌ی $x^2-(\sin\alpha)x-\frac{1}{2}\cos^2\alpha=0$

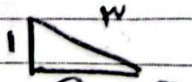
$x^2-(\sin\alpha)x-\frac{1}{2}\cos^2\alpha=0$

$\to \frac{4}{9}-\frac{2}{3}\sin\alpha-\frac{1}{2}\cos^2\alpha=0 \xrightarrow{\times 18} 8-12\sin\alpha-\underbrace{9\cos^2\alpha}_{-9(1-\sin^2\alpha)}=0$

۲ ✓ $\to 9\sin^2\alpha-12\sin\alpha-1=0 \xrightarrow{\Delta} \sin\alpha \to \frac{12+18}{18}>1 \to \times$

$\to \sin\alpha = \frac{12-18}{18} = -\frac{1}{3}$ ✓

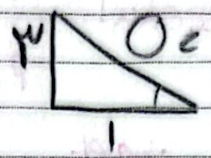
$\to 1+\tan^2\alpha = \frac{1}{\cos^2\alpha} = \frac{1}{\frac{8}{9}} = \boxed{\frac{9}{8}}$

$\sin\alpha = \frac{1}{3}$  $\to \cos\alpha = \frac{\sqrt{8}}{3}$

$\sin\alpha > 0$, $\cos\alpha < 0$

$\log(\sin\alpha) - \log(-\cos\alpha) = \log 3 \to \sin\alpha - \cos\alpha = ?$ (۳

$\hookrightarrow \log\left(\frac{\sin\alpha}{-\cos\alpha}\right) = \log(-\tan\alpha) = \log 3 \to$

$\tan\alpha = -3 \to$ 

$\to \sin\alpha = \frac{3}{\sqrt{10}}$, $\cos\alpha = -\frac{1}{\sqrt{10}}$

(نسبت رو مثبت بگیر!)

② ✓

$\to \sin\alpha - \cos\alpha = \frac{3}{\sqrt{10}} + \frac{1}{\sqrt{10}} = \frac{4}{\sqrt{10}} = \frac{4\sqrt{10}}{10} = \boxed{\frac{2\sqrt{10}}{5}}$

$\sin^4\theta + \cos^2\theta = \frac{4}{5} \to \cos^4\theta + \sin^2\theta = ?$ (۴

$\hookrightarrow \sin^4\theta + 1 - \sin^2\theta = \frac{4}{5} \to \underline{\sin^4\theta - \sin^2\theta + 1 = \frac{4}{5}}$

$\to \underset{\underset{(1-\sin^2\theta)^2}{\downarrow}}{\cos^4\theta} + \sin^2\theta = \sin^4\theta - 2\sin^2\theta + 1 + \sin^2\theta =$

$\underline{\sin^4\theta - \sin^2\theta + 1} = \boxed{\frac{4}{5}}$

② ✓

$\frac{3\sin x + \sin x\cos x}{2 - 2\cos^2 x} < 0$ و $\sin x\tan x - \frac{1}{\cos x} > 0 \to$ ناحیه؟ (۵

$(\sin x\cos x \neq 0)$

$\to \frac{\cancel{\sin x}(\cos x + 3)}{2(\underbrace{1-\cos^2 x}_{\sin^2 x})} = \frac{\overbrace{\cos x + 3}^{2 < \ \ < 4 \to \oplus}}{2\sin x} < 0 \to \sin x < 0$

I ربع ۳ یا ۴

② ✓

$\sin x \underbrace{\tan x}_{\frac{\sin x}{\cos x}} - \frac{1}{\cos x} > 0 \to \frac{\sin^2 x - 1}{\cos x} > 0 \to \frac{-\cos^2 x}{\cos x} > 0 \to$

$-\cos x > 0 \to$

II $\cos x < 0 \to$ ۲ یا ۳

$\mathrm{I} \cap \mathrm{II} \to \boxed{\text{ربع ۳}}$

$$\frac{2}{\sin x} + \frac{3}{\cos x} = 0 \rightarrow \tan x + \cot x = ?$$ (6

$(\sin x, \cos x \neq 0)$

$$\frac{2\cos x + 3\sin x}{\sin x \cos x} = 0 \rightarrow 2\cos x + 3\sin x = 0 \rightarrow$$

$$2\cos x = -3\sin x \rightarrow \frac{\sin x}{\cos x} = -\frac{2}{3} \rightarrow$$

$$\tan x + \cot x = -\frac{2}{3} - \frac{3}{2} = \boxed{\frac{-13}{6}}$$

(2) ✓

$$\frac{\sqrt{1+\sin 50^\circ}}{\sin 80^\circ + \sin 10^\circ} = ?$$ (7

$$* \; 1+\sin\theta = \left(\cos\frac{\theta}{2} + \sin\frac{\theta}{2}\right)^2$$

$$1 + 2 \times \frac{1}{2}\sin\theta = 1+\sin\theta$$

$$\frac{\sin 45^\circ + \sin 25^\circ}{\sin 80^\circ + \sin 10^\circ} \rightarrow = \sqrt{2}\sin(25+45) = \sqrt{2}\sin 70 = \sqrt{2}\cos 20^\circ$$

$$= 2\sin 30^\circ \cos 20^\circ = \cos 20^\circ$$

(2) ✓

$$\rightarrow \frac{\sqrt{2}\cos 20^\circ}{\cos 20^\circ} = \boxed{\sqrt{2}}$$

$$* \; \sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$(1 - 2\sin^2 5^\circ)(2\cos^2 10^\circ - 1)\left(\frac{1-\tan^2 20^\circ}{1+\tan^2 20^\circ}\right) = ?$$ (8

$$2(1-\cos 10) \qquad \cos 20 \qquad \cos 40 \qquad = \cos 40 \times \cos 20 \times \cos 10$$

$$\sin^2 5 = \frac{1-\cos 10}{2} \qquad \cos^2 10 = \frac{1+\cos 20}{2}$$

(2) ✓

$$\frac{1}{2}\sin 20 \quad \frac{1}{4}\sin 40 \quad \frac{1}{8}\sin 80$$

$$\times \frac{\sin 10}{\sin 10} \rightarrow \frac{\sin 10 \cos 10 \times \cos 20 \times \cos 40}{\sin 10} = \frac{\frac{1}{8}\cos 10}{\sin 10} = \boxed{\frac{\cot 10^\circ}{8}}$$

(۹ $\sin\alpha + ۳\cos\alpha = ۲ \rightarrow ۵\cos ۲\alpha - ۱۲\cos\alpha = ?$

$\sin\alpha = ۲ - ۳\cos\alpha \qquad \xrightarrow{(\;)^۲} \sin^۲\alpha + ۹\cos^۲\alpha + ۶\cos\alpha\sin\alpha = ۴$

$\rightarrow ۱ - \cos^۲\alpha + ۹\cos^۲\alpha + ۶\cos\alpha\,(۲ - ۳\cos\alpha) = ۴$

$\rightarrow -۱۰\cos^۲\alpha + ۱۲\cos\alpha = ۳$ ** ۲ ✓

$\rightarrow ۵\underbrace{\cos ۲\alpha}_{\cos^۲\alpha = \frac{۱+\cos ۲\alpha}{۲}} - ۱۲\cos\alpha = \underbrace{۵(۲\cos^۲\alpha - ۱) - ۱۲\cos\alpha}_{۱۰\cos^۲\alpha - ۵ - ۱۲\cos\alpha} = \boxed{-۸}$

$**\ \underbrace{۱۰\cos^۲\alpha - ۱۲\cos\alpha}_{-۳} - ۵$

(۱۰ $\sin B \times \sin C = \cos^۲\frac{A}{۲} \rightarrow \sin B \sin C = \frac{۱+\cos A}{۲}$

$\rightarrow \sin B \times \sin C = \frac{۱+\cos A}{۲} \rightarrow ۲\sin B \times \sin C = ۱ + \cos A$

$۲\left(\frac{۱}{۲}\left(\cos(B-C) - \cos(\pi - A)\right)\right) = \cos A + ۱$ ۲ ✓

$\cos(B-C) + \cos A = \cos A + ۱ \rightarrow \cos(\underbrace{B-C}_{=۰}) = ۱$

$\rightarrow B = C \rightarrow ۱۸۰ - ۱۴۴ = \boxed{A = ۳۶^\circ}$