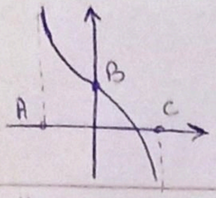
1.



$\tan(-2x+\frac{\pi}{4}) \quad AC = T = \frac{\pi}{|b|} = \frac{\pi}{2}$

$f(0) = \tan(\frac{\pi}{4}) = 1 = B \quad S = \frac{1}{2} \times AC \times OB = \frac{1}{2} \times \frac{\pi}{2} \times 1 = \boxed{\frac{\pi}{4}}$

2.

$x^2 - (\sin\alpha)x - \frac{1}{4}\cos^2\alpha = 0 \quad x = \frac{2}{3} \quad 1+\tan^2\alpha = \frac{1}{\cos^2\alpha} = \frac{1}{\frac{8}{9}} = \boxed{\frac{9}{8}}$

$\Rightarrow \frac{4}{9} - \frac{2}{3}(\sin\alpha) - \frac{1}{4}\cos^2\alpha = 0 \Rightarrow \frac{4}{9} - \frac{2}{3}(\sin\alpha) - \frac{1}{4}(1-\sin^2\alpha) = 0$

$\Rightarrow \frac{1}{4}\sin^2\alpha - \frac{2}{3}\sin\alpha + \underbrace{\frac{4}{9} - \frac{1}{4}}_{\frac{16-9}{36}} = 0 \Rightarrow 9\sin^2\alpha - 24\sin\alpha + 7 = 0 \Rightarrow (3\sin\alpha - 1)(3\sin\alpha - 7) = 0$

$\sin\alpha = \frac{1}{3}$ ق ق

$\sin\alpha = \frac{7}{3}$ غ ق ق

$\underbrace{\sin^2\alpha}_{\frac{1}{9}} + \cos^2\alpha = 1 \Rightarrow \cos^2\alpha = \frac{8}{9}$

3.

$\log(\sin\alpha) - \log(-\cos\alpha) = \log 3 \quad \log_b^a \begin{cases} a>0 \\ b>0 \\ b \neq 1 \end{cases} \Rightarrow \begin{cases} \sin\alpha > 0 \\ \cos\alpha < 0 \end{cases}$

$\log\frac{\sin\alpha}{-\cos\alpha} = \log 3 \Rightarrow \frac{\sin\alpha}{-\cos\alpha} = 3 \Rightarrow \sin\alpha = -3\cos\alpha \quad \sin^2\alpha + \cos^2\alpha = 1 \Rightarrow 10\cos^2\alpha = 1$

$\sin^2\alpha = 9\cos^2\alpha \quad \Rightarrow \cos\alpha = \frac{-\sqrt{10}}{10} \;,\; \sin\alpha = \frac{\sqrt{90}}{10}$

$\sin\alpha - \cos\alpha = \frac{\sqrt{90}}{10} + \frac{\sqrt{10}}{10} = \frac{3\sqrt{10}+\sqrt{10}}{10} = \frac{4\sqrt{10}}{10} = \frac{2\sqrt{10}}{5}$

4.

$\sin^4\alpha + \cos^2\alpha = \frac{4}{5} \Rightarrow \sin^4\alpha + 1 - \sin^2\alpha = \frac{4}{5} \Rightarrow \sin^4\alpha - \sin^2\alpha + \frac{1}{5} = 0$

$t^2 - t + \frac{1}{5} = 0 \quad t = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{1 \pm \frac{\sqrt{5}}{5}}{2}$

$\to \sin^2\alpha = \frac{5+\sqrt{5}}{10}$ ق ق $\to \cos^2\alpha = \frac{5-\sqrt{5}}{10}$

$\to \sin^2\alpha = \frac{5-\sqrt{5}}{10}$ ق ق $\to \cos^2\alpha = \frac{5+\sqrt{5}}{10}$

$\cos^4\alpha + \sin^2\alpha \xrightarrow{\text{حالت ۱}} \frac{25+5-10\sqrt{5}}{100} + \frac{5+\sqrt{5}}{10} = 0.8$

$\xrightarrow{\text{حالت ۲}} \frac{25+5+10\sqrt{5}}{100} + \frac{5-\sqrt{5}}{10} = 0.8$

$\Rightarrow \cos^4\alpha + \sin^2\alpha = 0.8$

5.

$\frac{3\sin\alpha + \sin\alpha\cos\alpha}{2-2\cos^2\alpha} < 0 \Rightarrow \frac{\sin\alpha(3+\cos\alpha)}{2(1-\cos^2\alpha)} < 0 \Rightarrow \frac{\sin\alpha\,\overbrace{(3+\cos\alpha)}^{+\text{همواره}}}{\underbrace{2\sin^2\alpha}_{+\text{همواره}}} < 0 \Rightarrow \sin\alpha < 0$

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$\sin\alpha\tan\alpha - \frac{1}{\cos\alpha} > 0 \Rightarrow \frac{\sin^2\alpha}{\cos\alpha} - \frac{1}{\cos\alpha} > 0 \Rightarrow \frac{-\cos^2\alpha}{\cos\alpha} > 0 \Rightarrow -\cos\alpha > 0 \Rightarrow \cos\alpha < 0$

6.

$\frac{2}{\sin x} = \frac{-3}{\cos x} \Rightarrow 2\cos x = -3\sin x \Rightarrow \cos x = \frac{-3}{2}\sin x$

$\sin^2 x + \cos^2 x = 1 \Rightarrow \sin^2 x + \frac{9}{4}\sin^2 x = 1 \Rightarrow \sin^2 x = \frac{4}{13} \;,\; \cos^2 x = \frac{9}{13} \quad \sin x = \frac{2}{\sqrt{13}} \quad \cos x = \frac{-3}{\sqrt{13}}$

$\tan x + \cot x = \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \frac{1}{\sin x\cos x} = \frac{1}{\frac{2}{\sqrt{13}} \times \frac{-3}{\sqrt{13}}} = \boxed{\frac{-13}{6}}$

$$\frac{\sqrt{1+\sin 50^\circ}}{\sin 50^\circ+\sin 10^\circ}=\frac{\sqrt{1+2\sin 25^\circ\cos 25^\circ}}{\sin 50^\circ+\sin 10^\circ}=\frac{\sqrt{\sin^2 25^\circ+\cos^2 25^\circ+2\sin 25^\circ\cos 25^\circ}}{\sin 50^\circ+\sin 10^\circ}=\frac{|\sin 25^\circ+\cos 25^\circ|}{\sin 50^\circ+\sin 10^\circ}$$ -7

$\sin 50^\circ = 2\sin 25^\circ\cos 25^\circ$ $\quad \sin 50^\circ=\sin(30^\circ+20^\circ)=\sin 30^\circ\cos 20^\circ+\cos 30^\circ\sin 20^\circ$

$\sin 10^\circ=\sin(30^\circ-20^\circ)=\sin 30^\circ\cos 20^\circ-\cos 30^\circ\sin 20^\circ$

$\sin 50^\circ+\sin 10^\circ=2\underbrace{\sin 30^\circ}_{\frac{1}{2}}\cos 20^\circ=\cos 20^\circ$

$\sin 25^\circ+\cos 25^\circ=\sqrt{2}\sin(25^\circ+45^\circ)=\sqrt{2}\sin 70^\circ=\sqrt{2}\cos 20^\circ$ $\quad \frac{\sqrt{2}\cos 20^\circ}{\cos 20^\circ}=\sqrt{2}$

$$\underbrace{(1-2\sin^2 5^\circ)}_{\cos 10^\circ}\underbrace{(2\cos^2 10^\circ-1)}_{\cos 20^\circ}\underbrace{\left(\frac{1-\tan^2 20^\circ}{1+\tan^2 20^\circ}\right)}_{\cos 40^\circ}\times\frac{\sin 10^\circ}{\sin 10^\circ}=\frac{1}{2}\sin 20^\circ\cos 20^\circ\cos 40^\circ=$$ -8

$\sin\alpha=\cos\left(\frac{\pi}{2}-\alpha\right)$

$\frac{1}{4}\sin 40^\circ\cos 40^\circ=\frac{1}{8}\sin 80^\circ=\frac{\sin 80^\circ}{8\times\sin 10^\circ}$ $\quad \frac{\cos 10^\circ}{8\times\sin 10^\circ}=\frac{1}{8}\cot 10^\circ$

$\sin\alpha+3\cos\alpha=2 \rightarrow \sin\alpha=2-3\cos\alpha$ -9

$\cos^2\alpha+\sin^2\alpha=1 \Rightarrow \cos^2\alpha+9\cos^2\alpha-12\cos\alpha+4=1 \Rightarrow$

$10\cos^2\alpha-12\cos\alpha+3=0 \Rightarrow \cos\alpha=\frac{12\pm 2\sqrt{6}}{20}=\frac{3\pm\sqrt{6}}{5}$ $\quad 10\cos^2\alpha-12\cos\alpha=-3$

$\cos 2\alpha=2\cos^2\alpha-1$ $\quad 5\cos 2\alpha-12\cos\alpha=\underbrace{10\cos^2\alpha-12\cos\alpha}_{-3}-5=-8$

-10