پریا شهرکی نیا — دوازدهم (A) — تکلیف حسابان ۱

۱ـ

$$y=\tan\left(-2x+\frac{\pi}{4}\right) \longrightarrow AC = \text{دوره تناوب} = \frac{\pi}{|b|} = \frac{\pi}{2}$$

$$B(0,y) \longrightarrow y=\tan\frac{\pi}{4} \to y=1 \to B(0,1)$$

$$S_{ABC}=\frac{AC\times BO}{2}=\frac{\frac{\pi}{2}\times 1}{2}=\frac{\pi}{4}$$

۲ـ

$$x^2-(\sin\alpha)x-\frac{1}{4}\underbrace{\cos^2\alpha}_{1-\sin^2\alpha}=0 \xrightarrow{x=\frac{2}{3}} \frac{4}{9}-\frac{2}{3}\sin\alpha-\frac{1}{4}+\frac{1}{4}\sin^2\alpha=0$$

$$\frac{1}{4}\sin^2\alpha-\frac{2}{3}\sin\alpha+\frac{7}{36} \longrightarrow \frac{-b\pm\sqrt{b^2-4ac}}{2a} \to \frac{\frac{2}{3}\pm\sqrt{\frac{4}{9}-\frac{7}{36}}}{\frac{1}{2}} \to$$

$$\to \frac{\frac{2}{3}\pm\sqrt{\frac{9}{36}}}{\frac{1}{2}}$$

$$\to \frac{\frac{2}{3}+\frac{3}{6}}{\frac{1}{2}}=\frac{7}{3}$$

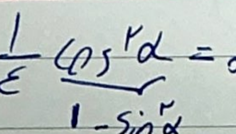
$$\to \frac{\frac{2}{3}-\frac{3}{6}}{\frac{1}{2}}=\frac{1}{3}$$

$$\longrightarrow -1\leq\sin\alpha\leq 1$$

$\frac{7}{3}$ غ ق ق

$\boxed{\frac{1}{3} \text{ ق ق}}$

$$\sin\alpha=\frac{1}{3} \to$$



$$\to \tan\alpha=\frac{1}{\sqrt{8}}$$

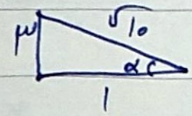
$$\tan^2\alpha+1=\frac{1}{8}+1=\frac{9}{8}$$

$$\log(\sin\alpha) - \log(-\cos\alpha) = \log ۳ \rightarrow \log \frac{\sin\alpha}{-\cos\alpha} = ۳ \rightarrow \tan\alpha = ۳ \rightarrow$$ -۳

$$\rightarrow \tan\alpha = -۳ \rightarrow \tan^{۲}\alpha + ۱ = \frac{۱}{\cos^{۲}\alpha} \rightarrow \cos^{۲}\alpha = \frac{۱}{۱۰} \rightarrow \log_b^a \rightarrow a > ۰ \rightarrow$$

$$\rightarrow \frac{\sin\alpha}{-\cos\alpha} > ۰ \rightarrow \tan\alpha < ۰ ,\ \sin\alpha > ۰ ,\ -\cos\alpha > ۰ \rightarrow \cos\alpha < ۰$$

$$\cos^{۲}\alpha = \frac{۱}{۱۰} \rightarrow \cos\alpha = -\frac{۱}{\sqrt{۱۰}} \rightarrow$$



$$\rightarrow \sin\alpha = \frac{۳}{\sqrt{۱۰}}$$

$$\sin\alpha - \cos\alpha = \frac{۳}{\sqrt{۱۰}} + \frac{۱}{\sqrt{۱۰}} = \frac{٤}{\sqrt{۱۰}} \times \frac{\sqrt{۱۰}}{\sqrt{۱۰}} = \frac{٤\sqrt{۱۰}}{۱۰} = \frac{۲\sqrt{۱۰}}{۵}$$

$$\sin^{٤}\theta + \cos^{۲}\theta = \frac{٤}{۵} \xrightarrow{\cos^{۲}\theta = ۱ - \sin^{۲}\theta} \sin^{٤}\theta - \sin^{۲}\theta + ۱ = \frac{٤}{۵}$$ -۴

$$\sin^{۲}\theta = a \rightarrow a^{۲} - a + \frac{۱}{۵} = ۰ \longrightarrow \cos^{٤}\theta + \sin^{۲}\theta = (۱-a)^{۲} + a \rightarrow$$

$$\rightarrow (۱-a)^{۲} + a = ۱ - ۲a + a^{۲} + a = \underbrace{a^{۲} - a + ۱}_{\sin^{٤}\theta - \sin^{۲}\theta + ۱} = \frac{٤}{۵}$$

$$\frac{۳\sin x + \sin x\cos x}{۲ - ۲\cos^{۲}x} < ۰ \rightarrow \frac{\cancel{\sin x}\,(۳ + \cos x)}{۲\underbrace{(۱ - \cos^{۲}x)}_{\sin^{۲}x}} < ۰ \rightarrow$$ -۵

$$\rightarrow \frac{\overbrace{۳ + \cos x}^{\text{همواره } +}}{۲\sin x} < ۰ \rightarrow \sin x < ۰ ,\ \sin x\tan x - \frac{۱}{\cos x} < ۰ \rightarrow$$

$$\rightarrow \frac{\overbrace{\sin^{۲}x - ۱}^{\text{همواره } -}}{\cos x} > ۰ \rightarrow \cos x < ۰ \longrightarrow \sin x < ۰ ,\ \cos x < ۰$$

ناحیه سوم

$$\frac{2}{\sin x}+\frac{3}{\cos x}=0 \rightarrow \frac{2\cos x+3\sin x}{\sin x\cos x}=0 \rightarrow 2\cos x+3\sin x=0 \quad 4$$

$$\rightarrow 3\sin x=-2\cos x \rightarrow \frac{\sin x}{\cos x}=\frac{-2}{3} \rightarrow \tan x=-\frac{2}{3},\ \cot x=-\frac{3}{2}$$

$$\tan x+\cot x=-\frac{2}{3}-\frac{3}{2}=\frac{-4-9}{6}=\frac{-13}{6}$$

$$\frac{\sqrt{1+\sin 50}}{\sin 50+\sin 10} \xrightarrow[1=\sin^2 25+\cos^2 25]{\sin 50=2\sin 25\cos 25} \frac{\sqrt{\sin^2 25+\cos^2 25+2\sin 25\cos 25}}{\sin(20+30)+\sin(30-20)}=$$

$$=\frac{\sqrt{(\sin 25+\cos 25)^2}}{\sin 20\cos 30+\sin 30\cos 20+\sin 30\cos 20-\sin 20\cos 30} \rightarrow$$

$$\frac{\sin 25+\cos 25}{2\sin 30\underbrace{\cos 20}_{\sin 70}} \xrightarrow{\frac{a}{|a|}\sqrt{a^2+b^2}\sin(x+\alpha),\ \tan\alpha=\frac{b}{a}\rightarrow\alpha=45^\circ} \frac{\sqrt{2}\sin(\overbrace{25+45}^{70})}{2\sin 30\sin 70} \rightarrow$$

$$\rightarrow \frac{\sqrt{2}\,\cancel{\sin 70}}{2\times\frac{1}{2}\times\cancel{\sin 70}}=\sqrt{2}$$

۸

$$\left(1-2\sin^2 5^\circ\right)\left(2\cos^2 10^\circ-1\right)\left(\frac{1-\tan^2 20}{1+\tan^2 20}\right)$$

$$1-2\sin^2 5^\circ \to 1-2\sin^2 5 = 1-2+2\cos^2 5 = 2\cos^2 5-1 = 1+\cos 10-1 = \cos 10^\circ$$

$$\frac{1-\tan^2 20}{1+\tan^2 20} \to \cos 2\alpha = \frac{1-\tan^2\alpha}{1+\tan^2\alpha} \to \frac{1-\tan^2 20}{1+\tan^2 20} = \cos 40^\circ$$

$$2\cos^2 10-1 \to 1+\cos 20-1 = \cos 20^\circ$$

$$\cos 10\times\cos 20\times\cos 40\times\frac{\sin 10}{\sin 10} = \frac{\frac{1}{2}\sin 20\cos 20\cos 40}{\sin 10} =$$

$$= \frac{\frac{1}{4}\sin 40\cos 40}{\sin 10} = \frac{\frac{1}{8}\sin 80}{\sin 10} \xrightarrow{\sin 80=\cos 10} \frac{1}{8}\,\frac{\cos 10}{\sin 10} = \frac{1}{8}\cot 10$$

۹

$$\sin\alpha+3\cos\alpha=2 \xrightarrow{\sin\alpha=\sqrt{1-\cos^2\alpha}} \sqrt{1-\cos^2\alpha}+3\cos\alpha=2$$

$$\sqrt{1-\cos^2\alpha}=2-3\cos\alpha \xrightarrow{\text{به توان ۲}} 1-\cos^2\alpha = 4+9\cos^2\alpha-12\cos\alpha \to$$

$$\to 10\cos^2\alpha-12\cos\alpha+3=0$$

$$5\cos 2\alpha-12\cos\alpha \xrightarrow{\cos 2\alpha=2\cos^2\alpha-1} 5(2\cos^2\alpha-1)-12\cos\alpha \to$$

$$\to \overbrace{10\cos^2\alpha-12\cos\alpha}^{-3}-5 = -8$$

ما

$$\sin\hat{B}\sin\hat{C} = \cos^2\frac{\hat{A}}{2} \quad , \quad \hat{B} = 72^\circ \qquad \hat{A} = ?$$

$$\cos^2\frac{A}{2} \longrightarrow \frac{1+\cos A}{2}$$

فرمول ضرب به جمع

$$\sin B \sin C = \frac{1}{2}\left(\cos(B-C) + \cos A\right) = \frac{1+\cos A}{2}$$

$$\cos(B-C) = 1 \qquad B - C = 0 \longrightarrow B = C = 72^\circ$$

$$\hat{A} + \hat{B} + \hat{C} = 180 \longrightarrow \hat{A} = 180 - 144 = 36^\circ$$